University of Debrecen
Faculty of Science and Technology
Institute of Mathematics

APPLIED MATHEMATICS MSC PROGRAM

2020

TABLE OF CONTENTS

DEAN`S WELCOME

Welcome to the Faculty of Science and Technology!

This is an exciting time for you, and I encourage you to take advantage of all that the Faculty of Science and Technology UD offers you during your bachelor’s or master's studies. I hope that your time here will be both academically productive and personally rewarding

Being a regional centre for research, development and innovation, our Faculty has always regarded training highly qualified professionals as a priority. Since the establishment of the Faculty in 1949, we have traditionally been teaching and working in all aspects of Science and have been preparing students for the challenges of teaching. Our internationally renowned research teams guarantee that all students gain a high quality of expertise and knowledge. Students can also take part in research and development work, guided by professors with vast international experience.

While proud of our traditions, we seek continuous improvement, keeping in tune with the challenges of the modern age. To meet the demand of the job market for professionals, we offer engineering courses with a strong scientific basis, thus expanding our training spectrum in the field of technology. Based on the fruitful collaboration with our industrial partners, recently, we successfully introduced dual-track training programmes in our constantly evolving engineering courses.

We are committed to providing our students with valuable knowledge and professional work experience, so that they can enter the job market with competitive degrees. To ensure this, we maintain a close relationship with the most important national and international companies. The basis for our network of industrial relationships are in our off-site departments at various different companies, through which market participants - future employers - are also included in the development and training of our students.

Prof. dr. Ferenc Kun
Dean

UNIVERSITY OF DEBRECEN

Date of foundation: 1912 Hungarian Royal University of Sciences, 2000 University of Debrecen

Legal predecessors: Debrecen University of Agricultural Sciences; Debrecen Medical University; Wargha István College of Education, Hajdúböszörmény; Kossuth Lajos University of Arts and Sciences

Legal status of the University of Debrecen: state university

Founder of the University of Debrecen: Hungarian State Parliament

Supervisory body of the University of Debrecen: Ministry of Education

Number of Faculties at the University of Debrecen: 14

Faculty of Agricultural and Food Sciences and Environmental Management

Faculty of Child and Special Needs Education

Faculty of Dentistry

Faculty of Economics and Business

Faculty of Engineering

Faculty of Health

Faculty of Humanities

Faculty of Informatics

Faculty of Law

Faculty of Medicine

Faculty of Music

Faculty of Pharmacy

Faculty of Public Health

Faculty of Science and Technology

Number of students at the University of Debrecen: 28593

Full time teachers of the University of Debrecen: 1541

208 full university professors and 1201 lecturers with a PhD.

FACULTY OF SCIENCE AND TECHNOLOGY

The Faculty of Science and Technology is currently one of the largest faculties of the University of Debrecen with about 3000 students and more than 200 staff members. The Faculty has got 6 institutes: Institute of Biology and Ecology, Institute of Biotechnology, Institute of Chemistry, Institute of Earth Sciences, Institute of Physics and Institute of Mathematics. The Faculty has a very wide scope of education dominated by science and technology (10 Bachelor programs and 12 Master programs), additionally it has a significant variety of teachers' training programs. Our teaching activities are based on a strong academic and industrial background, where highly qualified teachers with a scientific degree involve student in research and development projects as part of their curriculum. We are proud of our scientific excellence and of the application-oriented teaching programs with a strong industrial support. The number of international students of our faculty is continuously growing (currently ~800 students). The attractiveness of our education is indicated by the popularity of the Faculty in terms of incoming Erasmus students, as well.

THE ORGANIZATIONAL STRUCTURE OF THE FACULTY

Dean: Prof. Dr. Ferenc Kun, University Professor
E-mail: ttkdekan@science.unideb.hu

Vice Dean for Educational Affairs: Prof. Dr. Gábor Kozma, University Professor
E-mail: kozma.gabor@science.unideb.hu

Vice Dean for Scientific Affairs: Prof. Dr. Sándor Kéki, University Professor
E-mail: keki.sandor@science.unideb.hu

Consultant on Economic Affairs: Dr. Sándor Alex Nagy, Associate Professor
E-mail: nagy.sandor.alex@science.unideb.hu

Consultant on External Relationships: Prof. Dr. Attila Bérczes, University Professor
E-mail: berczesa@science.unideb.hu

Quality Assurance Coordinator: Dr. Zsolt Radics, Assistant Professor
E-mail: radics.zsolt@science.unideb.hu

Dean's Office
Head of Dean's Office: Ms. Katalin Tóth
E-mail: toth.kata@science.unideb.hu

Registrar's Office
Registrar: Ms. Ildikó Kerekes
E-mail: kerekes.ildiko@science.unideb.hu

English Program Officer: Mr. Imre Varga
Address: 4032 Egyetem tér 1., Chemistry Building, A/101
E-mail: vargaimre@unideb.hu

DEPARTMENTS OF INSTITUTE OF MATHEMATICS

Department of Algebra and Number Theory (home page: http://math.unideb.hu/algebra/en)
4032 Debrecen, Egyetem tér 1, Geomathematics Building

Name	Position	E-mail	room
Mr. Prof. Dr. Attila Bérczes	University Professor, Head of Department	berczesa@science.unideb.hu	M415
Mr. Prof. Dr. István Gaál	University Professor	gaal.istvan@unideb.hu	M419
Mr. Prof. Dr. Lajos Hajdu	University Professor, Director of Institute	hajdul@science.unideb.hu	M416
Mr. Prof. Dr. Ákos Pintér	University Professor	apinter@science.unideb.hu	M417
Mr. Dr. Szabolcs Tengely	Associate Professor	tengely@science.unideb.hu	M415
Mr. Dr. András Pongrácz	Associate Professor	pongracz.andras@science.unideb.hu	M406
Mr. Dr. András Bazsó	Assistant Professor	bazsoa@science.unideb.hu	M407
Mr. Dr. Gábor Nyul	Assistant Professor	gnyul@science.unideb.hu	M405
Mr. Dr. István Pink	Assistant Professor	pinki@science.unideb.hu	M405
Mrs. Dr. Nóra Györkös-Varga	Assistant Lecturer	nvarga@science.unideb.hu	M417
Mrs. Dr. Eszter Szabó-Gyimesi	Assistant Lecturer	gyimesie@science.unideb.hu	M404
Mr. Dr. Márton Szikszai	Assistant Lecturer	szikszai.marton@science.unideb.hu	M407
Ms. Tímea Arnóczki	PhD student	arnoczki.timea@science.unideb.hu	M404
Mr. Csanád Bertók	Assistant Research Fellow	bertok.csanad@inf.unideb.hu	M408
Ms. Judit Ferenczik	Assistant Research Fellow	jferenczik@science.unideb.hu	M407
Ms. Gabriella Rácz	PhD student	racz.gabriella@science.unideb.hu	M404
Mr. László Remete	PhD student	remete.laszlo@science.unideb.hu	M404

Department of Analysis (home page: http://math.unideb.hu/analizis/en)
4032 Debrecen, Egyetem tér 1, Geomathematics Building

Name	Position	E-mail	room
Mr. Dr. Zoltán Boros	Associate Professor Head of Department	zboros@science.unideb.hu	M326
Mr. Prof. Dr. Zsolt Páles	University Professor,	pales@science.unideb.hu	M321
Mr. Prof. Dr. György Gát	University Professor	gat.gyorgy@science.unideb.hu	M324
Mr. Prof. Dr. László Székelyhidi	University Professor	szekely@science.unideb.hu	M327
Mr. Dr. Mihály Bessenyei	Associate Professor	besse@science.unideb.hu	M326
Mrs. Dr. Eszter Novák-Gselmann	Associate Professor	gselmann@science.unideb.hu	M325

Ms. Dr. Borbála Fazekas	Assistant Professor	borbala.fazekas@science.unideb.hu	M325
Mr. Dr. Rezső László Lovas	Assistant Professor	lovas@science.unideb.hu	M330
Ms. Dr. Fruzsina Mészáros	Assistant Professor	mefru@science.unideb.hu	M325
Mr. Dr. Gergő Nagy	Assistant Professor	nagyg@science.unideb.hu	M323
Mr. Tibor Kiss	Assistant Lecturer	kiss.tibor@science.unideb.hu	M322
Mr. Gábor Lucskai	PhD student	gabor.lucskai@science.unideb.hu	M322

Department of Geometry (home page: http http://math.unideb.hu/geometria/en)
4032 Debrecen, Egyetem tér 1, Geomathematics Building

Name	Position	E-mail	room
Mr. Dr. Zoltán Muzsnay	Associate Professor, Head of Department	muzsnay@science.unideb.hu	M305
Ms. Dr. Ágota Figula	Associate Professor	figula@science.unideb.hu	M303
Mrs. Dr. Eszter Herendiné Kónya	Associate Professor	eszter.konya@science.unideb.hu	M307
Mr. Dr. Zoltán Kovács	Associate Professor	kovacsz@science.unideb.hu	M303
Mr. Dr. László Kozma	Associate Professor	kozma@unideb.hu	M306
Mr. Dr. Csaba Vincze	Associate Professor, Deputy Director of Institute	csvincze@science.unideb.hu	M304
Mr. Dr. Tran Quoc Binh	Senior Research Fellow	binh@science.unideb.hu	M305
Mr. Dr. Zoltán Szilasi	Assistant Professor	szilasi.zoltan@science.unideb.hu	M329
Mr. Dr. Ábris Nagy	Assistant Lecturer	abris.nagy@science.unideb.hu	M304
Mr. Balázs Hubicska	PhD student	hubicska.balazs@science.unideb.hu	M329

ACADEMIC CALENDAR

General structure of the academic semester (2 semesters/year):

Study period	1st week	Registration*	1 week
	2nd – 15th week	Teaching period	14 weeks
Exam period	directly after the study period	Exams	7 weeks

*Usually, registration is scheduled for the first week of September in the fall semester, and for the first week of February in the spring semester.

For further information please check the following link:
https://www.edu.unideb.hu/tartalom/downloads/University_Calendars_2020_21/20_21_Science.pdf

THE APPLIED MATHEMATICS MSC PROGRAM

Information about the Program

Name of BSc Program:	Applied Mathematics MSc Program
Specialization available:	
Field, branch:	Science
Qualification:	Applied Mathematician
Mode of attendance:	Full-time
Faculty, Institute:	Faculty of Science and Technology Institute of Mathematics
Program coordinator:	Prof. Dr. Ákos Pintér, University Professor
Duration:	4 semesters
ECTS Credits:	120

Objectives of the MSc program:

The aim of the Applied Mathematics MSc program is to train applied mathematicians who have research-level knowledge and modelling experience that makes them capable of solving problems in daily life practice. They are open to receive new results of their professional field. They are able to model and solve daily life problems and manage to implement solutions. They are prepared to continue to study in a PhD program.

Professional competences to be acquired

An Applied Mathematician:

a) Knowledge:
- He/she knows the methods of mathematical sciences, regarding theories in the fields of algorithms, applied analysis, discrete mathematics, operations research, probability theory and mathematical statistics, both at a system level and in context
- He/she knows the results of applied mathematics in context, regarding theories in the fields of algorithms, applied analysis, discrete mathematics, operations research, probability theory and mathematical statistics.
- He/she knows the deeper and more comprehensive correlations between the subdisciplines of applied mathematics, and how these fields interrelate and build upon each other.
- He/she has a knowledge of abstract mathematical thinking, and that of abstract mathematical terms and concepts.
- He/she has an appropriate knowledge of computer science and information technology necessary for the formulation and simulation of applied mathematical models.
- He/she knows the fundamentals of the theory of differential equations and approximating calculations, as well as, their most important applications in the modelling of natural, technical and economic phenomena.
- He/she knows the fundamentals of the modern theory of probability theory and mathematical statistics.

- He/she knows the fundamentals of coding theory and cryptography, the theoretical background and applicability of the codes and encryptions most commonly used in practice.
- He/she knows the theoretical background of approximating problems.
- He/she knows how to use the most important mathematical and statistical software packages, as well as, he/she is aware of their mathematical background and the limits of their applicability.
- He/she has a basic knowledge of micro- and macro-economics, and that of financial literacy.
- He/she knows the different procedures of modelling stochastic phenomena and processes.
- He/she is aware of the mathematical theory of stochastic and financial processes, time series, venture processes, life insurance and non-life insurance.
- He/she knows the mathematical analyses and models of financial processes and insurance issues.

b) Abilities:
- He/she is capable of applying the methods of mathematical sciences regarding theories in the fields of algorithms, applied analysis, discrete mathematics, operations research, probability theory and mathematical statistics.
- He/she is capable of establishing the mathematical models of phenomena observed in the surrounding world, as well as, of using the results of modern mathematics to explain and describe these phenomena.
- He/she is capable of abstraction, that is, capturing interrelations observed in daily life practice on an abstract level.
- He/she is capable of creatively combining and using his/her knowledge acquired in different application areas of mathematics to solve problems emerging in animate and inanimate nature, in the world of engineering and information technology, and in economic and financial life.
- He/she is capable of understanding complicated systems emerging in nature, engineering and economic life, of executing a mathematical analysis and modelling of them, and the ability to prepare decision-making processes.
- He/she is capable of understanding the internal mechanisms underlying problems, as well as, designing tasks and executing them at a high level.
- He/she is capable of formulating optimisation problems possibly underlying everyday decision situations, as well as, communicating the related conclusions to non-professionals.
- He/she is capable of executing calculation tasks emerging in nature, engineering and economic life, using computational tools and methods.
- He/she is capable of recognising tasks that require long series of computations and huge storage capacity, and of analysing alternative approaches.
- He/she is capable of clearly presenting mathematical results and arguments, as well as the related conclusions and is capable of professional communication.
- He/she is capable of competently interpreting the problems of his/her own professional field both for professionals and non-professionals.

c) Attitude:
- He/she aspires to get acquainted with new results of applied mathematics.
- He/she aspires to apply the results of applied mathematics as widely as possible.
- With the help of his/her knowledge acquired in applied mathematics, he/she aspires to distinguish between scientifically well-established (exact) statements and inadequately substantiated ones in his/her own professional field.
- He/she aspires to recognize further correlations between modern options of application in the field of applied mathematics, to synthetize and evaluate them at a high level and with scientific justification, using the tools of his/her own profession.

- He/she is receptive and open to adapting the different ways of reasoning, methods and concepts acquired in the field of applied mathematics to new fields of application, as well as, to achieving new results.
- He/she continuously aspires to enhance the scope of his/her knowledge, to learn new mathematical competencies.

d) Autonomy and responsibility:
- He/she responsibly, self-critically and realistically measures his/her knowledge acquired in the field of applied mathematics.
- With the help of his/her critical attitude and the system thinking skills he/she acquired, he/she participates in group work with responsibility, and if needed, cooperates with experts from professional fields other than his/hers.
- With the help of his/her high-level knowledge of applied mathematics, he/she makes an independent selection as to which methods and procedures he/she will use when solving different application problems.
- In his/her research activities, as well as, in mathematical applications, he/she considers it important to execute these practices in line with the highest ethical standards.
- He/she is aware, on the one hand, of the importance of mathematical thinking and precise conceptualization, and on the other hand, of the limits of applying mathematical models; thus he/she formulates his/her opinion on that basis.
- When applying mathematics, he/she responsibly represents his/her opinion formulated on the basis of his/her acquired knowledge.

Completion of the MSc Program

The Credit System

Majors in the Hungarian Education System have generally been instituted and ruled by the Act of Parliament under the Higher Education Act. The higher education system meets the qualifications of the Bologna Process that defines the qualifications in terms of learning outcomes: statements of what students know and can do on completing their degrees. In describing the cycles, the framework uses the European Credit Transfer and Accumulation System (ECTS).
ECTS was developed as an instrument of improving academic recognition throughout the European Universities by means of effective and general mechanisms. ECTS serves as a model of academic recognition, as it provides greater transparency of study programs and student achievement. ECTS in no way regulates the content, structure and/or equivalence of study programs.
Regarding each major the Higher Education Act prescribes which professional fields define a certain training program. It contains the proportion of the subject groups: natural sciences, economics and humanities, subject-related subjects and differentiated field-specific subjects.

During the program students have to complete a total amount of 120 credit points. It means approximately 30 credits per semester. The curriculum contains the list of subjects (with credit points) and the recommended order of completing subjects which takes into account the prerequisite(s) of each subject. You can find the recommended list of subjects/semesters in chapter “Model Curriculum of Applied Mathematics MSc Program”.

Model Curriculum of Applied Mathematics MSc Program

	semesters				ECTS credit points	evaluation
	1.	2.	3.	4.		
	contact hours, types of teaching (l – lecture, p – practice), credit points					
Basics Students having a BSc degree in Mathematics are granted exemption from these subjects. Students having degree in other subjects have to put in a credit-acceptance form. The Institue of Mathematics will decide what basic subjects the students will have to learn.						
Introduction to modern algebra Dr. Pongrácz András	28l/3cr. 28p/2cr.				3+2	exam mid-semester grade
Selected topics in geometry Dr. Kozma László	28l/3cr. 28p/2cr.				3+2	exam, mid-semester grade
Operation research Dr. Mészáros Fruzsina	28l/3cr. 28p/2cr.				3+2	exam mid-semester grade
Probability theory Dr. Fazekas István	28l/3cr. 28p/2cr.				3+2	exam mid-semester grade
Advanced prof subject group						
Graph Theory and Applications Dr. Nyul Gábor	28l/3cr. 28p/2cr.				3+2	exam mid-semester grade
Algorithms in mathematics Dr. Bérczes Attila		28l/3cr. 28p/2cr.			3+2	exam mid-semester grade
Convex optimization Dr. Bessenyei Mihály	28l/3cr. 28p/2cr.				3+2	exam mid-semester grade
Discrete Optimization Dr. Nyul Gábor		28l/3cr. 28p/2cr.			3+2	exam mid-semester grade
Applications of ordinary differential equations Dr. Novák-Gselmann Eszter	.		28l/3cr. 28p/2cr.		3+2	exam mid-semester grade
Partial differential equations Dr. Fazekas Borbála				28l/3cr. 28p/2cr.	3+2	exam mid-semester grade
Stochastic processes Dr. Szokol Patrícia		28l/3cr. 28p/2cr.			3+2	exam mid-semester grade
Multivariate analysis Dr. Baran Sándor			28l/3cr. 28p/2cr.		3+2	exam mid-semester grade
Option pricing Dr. Gáll József	28l/3cr. 28p/2cr.				3+2	exam mid-semester grade
Financial mathematics I Dr. Gáll József		28l/3cr. 28p/2cr.			3+2	exam mid-semester grade
Introduction to finance Dr. Gáll József	28l/3cr. 28p/2cr.				5	exam

Microeconomics Dr. Kapás Judit		28l/3cr. 28p/2cr.			5	exam
Econometrics Dr. Baran Sándor			28l/3cr. 14p/2cr.		4	exam
Financial accounting Dr. Tóth Kornél				28l/3cr. 28p/2cr.	5	exam
Game theory Dr. Boros Zoltán				28l/3cr. 28p/2cr.	5	exam
Elective courses The required credits points of elective subjects depend on how many subjects are accepted from the Basics. (The student has to learn subjects from elective courses for the same amount of credit points that is accepted from the Basics.)						
Macroeconomics Dr. Czeglédi Pál		.	28l/3cr. 28p/2cr.		5	exam
Insurance mathematics Dr. Barczy Mátyás		28l/3cr. (or semester 4)			3	exam
Financial mathematics II Dr. Gáll József			28l/3cr.		3	exam
Finite Geometries and Coding Theory Dr. Szilasi Zoltán		28l/3cr. 28p/2cr. (or semester 4)			3+2	exam mid-semester grade
Fourier series Dr. Gát György			28l/3cr. 14p/1cr.		4	exam

Thesis I.			10 cr.		10	mid-semester grade
Thesis II.				10 cr.	10	mid-semester grade

Optional courses						
Free optional courses					***6 cr***	

Work and Fire Safety Course

According to the Rules and Regulations of University of Debrecen a student has to complete the online course for work and fire safety. Registration for the course and completion are necessary for graduation. For MSc students the course is only necessary only if BSc diploma has been awarded outside of the University of Debrecen.
Registration in the Neptun system by the subject: MUNKAVEDELEM
Students have to read an online material until the end to get the signature on Neptun for the completion of the course. The link of the online course is available on webpage of the Faculty.

Physical Education

According to the Rules and Regulations of University of Debrecen a student has to complete Physical Education courses at least in one semester during his/her Master's training. Our University offers a wide range of facilities to complete them. Further information is available from the Sport Centre of the University, its website: http://sportsci.unideb.hu.

Pre-degree Certification

A pre-degree certificate is issued by the Faculty after completion of the master's (MSc) program. The pre-degree certificate can be issued if the student has successfully completed the study and exam requirements as set out in the curriculum, the requirements relating to Physical Education as set out in Section 10 in Rules and Regulations, internship (mandatory) – with the exception of preparing thesis – and gained the necessary credit points (120). The pre-degree certificate verifies (without any mention of assessment or grades) that the student has fulfilled all the necessary study and exam requirements defined in the curriculum and the requirements for Physical Education. Students who obtained the pre-degree certificate can submit the thesis and take the final exam.

Thesis

Students have to choose a topic for their thesis in the 2nd semester. They have to write it in two semesters. The thesis should be about 25–40 pages long and using the LaTeX document preparation system is recommended. The cover page has to contain the name of the institute, the title of the thesis, the name and the degree program of the student, the name and the university rank of the supervisor. Besides the detailed discussion of the topic, the thesis should contain an introduction, a table of contents and a bibliography. Further formal requirements and recommended style files can be found on the homepage of the Institute of Mathematics. The thesis has to be defended in the final exam.

Final Exam

The final exam is an oral exam before a committee designated by the Director of the Institute of Mathematics and approved by the leaders of the Faculty of Science and Technology. The process of the final exam is the same in both cases of the student's specialization. The questions of the final exam comprise the compulsory courses of the Applied Mathematics MSc Program.

The list of exam questions consists of two parts: questions from the core material, and questions pertaining to the student's specialization. Students draw a random question from the entire list, and after a certain preparation period, give an account on it. After this, the committee chooses a small item from one of the other type of questions, and after a preparation period the student gives an account on this as well. The committee gives a single mark for the student's answers in the final exam.

Final Exam Board

Board chair and its members are selected from the acknowledged internal and external experts of the professional field. Traditionally, it is the chair and in case of his/her absence or indisposition the vice-chair who will be called upon, as well. The board consists of – besides the chair – at least two members (one of them is an external expert), and questioners as required. The mandate of a Final Examination Board lasts for one year.

Repeating a failed Final Exam

If any part of the final exam is failed it can be repeated according to the rules and regulations. A final exam can be retaken in the forthcoming final exam period. If the Board qualified the Thesis unsatisfactory a student cannot take the final exam and he has to make a new thesis. A repeated final exam can be taken twice on each subject.

Diploma

The diploma is an official document decorated with the coat of arms of Hungary which verifies the successful completion of studies in the Applied Mathematics Master Program. It contains the following data: name of HEI (higher education institution); institutional identification number; serial number of diploma; name of diploma holder; date and place of his/her birth; level of qualification; training program; specialization; mode of attendance; place, day, month and year issued. Furthermore, it has to contain the rector's (or vice-rector's) original signature and the seal of HEI. The University keeps a record of the diplomas issued.

In Applied Mathematics Master Program the diploma grade is calculated as the average grade of the results of the followings:

- Weighted average of the overall studies at the program (A)
- Average of grades of the thesis and its defense given by the Final Exam Board (B)
- Average of the grades received at the Final Exam for the two subjects (C)

Diploma grade = (A + B + C)/3

Classification of the award on the bases of the calculated average:

Excellent	4.81 – 5.00
Very good	4.51 – 4.80
Good	3.51 – 4.50
Satisfactory	2.51 – 3.50
Pass	2.00 – 2.50

Course Descriptions of Applied Mathematics MSc Program

Title of course: Introduction to modern algebra **Code**: TTMME0101	**ECTS Credit points:** 3

Type of teaching, contact hours
- lecture: 2 hours/week
- practice: -
- laboratory: -

Evaluation: exam

Workload (estimated), divided into contact hours:
- lecture: 28 hours/week
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 62 hours

Total: 90 hours

Year, semester: 1st year, 1st semester

Its prerequisite(s): -

Further courses built on it: -

Topics of course

Sylow's theorems. Semidirect product. Maximal subgroups of p-groups are normal of index p. Characteristic subgroup, commutator. Solvable groups and their basic properties. Alternating group is simple if acting on at least 5 points. Free groups, generators, relations, Dyck's theorem. Necessary and sufficient condition for a ring to be a unique factorization domain, ascending and descending chain conditions. Field of fractions. Artinian and Noetherian rings, Hilbert's basis theorem. Algebras, minimal polynomial over algebras, Frobenius's theorem. Splitting field, existence, uniqueness, algebraic closure existence, uniqueness. Normal extensions, extensions of perfect fields are simple. Galois theory. Fundamental theorem of algebra. Compass and ruler constructions. Theorem of Abel and Ruffini, Casus Irreducibilis is unavoidable for degree three polynomials.

Literature

Compulsory:

-

Recommended:

John B. Fraleigh: A first course in abstract algebra, Addison-Wesley Publishing Company, 1989.
Derek J. S. Robinson: A course in the theory of groups, Springer-Verlag, 1980.

Schedule:

1st week

Sylow's theorems. Semidirect products.

2nd week

Maximal subgroups of p-groups are normal of index p. Characteristic subgroup, commutator. Fundamental theorem of finite Abelian groups.

3rd week

Isomorphism theorems. Solvable groups and their basic properties. Alternating group is simple if acting on at least 5 points.

4th week

Free groups, generators, relations, Dyck's theorem.

5th week

Necessary and sufficient condition for a ring to be a unique factorization domain, ascending and descending chain conditions.

6th week

Field of fractions. Artinian and Noetherian rings, Hilbert's basis theorem.

7th week

First test.

8th week

Algebras, minimal polynomial over algebras, Frobenius' theorem.

9th week

Splitting field, existence, uniqueness, algebraic closure existence, uniqueness.

10th week

Normal extensions, finite extensions of perfect fields are simple.

11th week

Fundamental theorem of Galois theory.

12th week

Fundamental theorem of algebra. Compass and straightedge constructions.

13th week

Theorem of Abel and Ruffini, Casus Irreducibilis is unavoidable for degree three polynomials.

14th week

Second test.

Requirements:

- for a signature

If the student fail the course TTMMG0101, then the signature is automatically denied.

- for a grade

The course ends in oral examination. The grade is given according to the following table:

Total Score (%)	Grade
0 – 49	fail (1)
50 – 59	pass (2)
60 – 69	satisfactory (3)
70 – 79	good (4)
80 – 100	excellent (5)

-an offered grade:

It is not possible to obtain an offered grade in this course.

Person responsible for course: Dr. András Pongrácz, assistant professor, PhD

Lecturer: Dr. András Pongrácz, assistant professor, PhD

Title of course: Introduction to modern algebra **Code**: TTMMG0101	**ECTS Credit points:** 2

Type of teaching, contact hours
- lecture: -
- practice: 2 hours/week
- laboratory: -

Evaluation: mid-semester grade

Workload (estimated), divided into contact hours:
- lecture: -
- practice: 28 hours
- laboratory: -
- home assignment: 32 hours
- preparation for the exam: -

Total: 60 hours

Year, semester: 1st year, 1st semester

Its prerequisite(s): -

Further courses built on it: -

Topics of course

Sylow's theorems. Semidirect product. Maximal subgroups of p-groups are normal of index p. Characteristic subgroup, commutator. Solvable groups and their basic properties. Alternating group is simple if acting on at least 5 points. Free groups, generators, relations, Dyck's theorem. Necessary and sufficient condition for a ring to be a unique factorization domain, ascending and descending chain conditions. Field of fractions. Artinian and Noetherian rings, Hilbert's basis theorem. Algebras, minimal polynomial over algebras, Frobenius's theorem. Splitting field, existence, uniqueness, algebraic closure existence, uniqueness. Normal extensions, extensions of perfect fields are simple. Galois theory. Fundamental theorem of algebra. Compass and ruler constructions. Theorem of Abel and Ruffini, Casus Irreducibilis is unavoidable for degree three polynomials.

Literature

Compulsory:

-

Recommended:

John B. Fraleigh: A first course in abstract algebra, Addison-Wesley Publishing Company, 1989.

Derek J. S. Robinson: A course in the theory of groups, Springer-Verlag, 1980.

Schedule:

1st week

Sylow's theorems. Semidirect products.

2nd week

Maximal subgroups of p-groups are normal of index p. Characteristic subgroup, commutator. Fundamental theorem of finite Abelian groups.

3rd week

Isomorphism theorems. Solvable groups and their basic properties. Alternating group is simple if acting on at least 5 points.

4th week

Free groups, generators, relations, Dyck's theorem.

5th week

Necessary and sufficient condition for a ring to be a unique factorization domain, ascending and descending chain conditions.

6th week

Field of fractions. Artinian and Noetherian rings, Hilbert's basis theorem.

7th week

Students can ask questions and get an overview on the subject material in all topics prior to the first test.

8th week

Algebras, minimal polynomial over algebras, Frobenius' theorem.

9th week

Splitting field, existence, uniqueness, algebraic closure existence, uniqueness.

10th week

Normal extensions, finite extensions of perfect fields are simple.

11th week

Fundamental theorem of Galois theory.

12th week

Fundamental theorem of algebra. Compass and straightedge constructions.

13th week

Theorem of Abel and Ruffini, Casus Irreducibilis is unavoidable for degree three polynomials.

14th week

Students can ask questions and get an overview on the subject material in all topics prior to the second test.

Requirements:

- for a signature

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

- for a grade

The preliminary requirement to pass the course is to obtain at least 51 percent of total points from short tests written on a once a week basis with the exception of the 1st, 7th and 14th week.

The course is evaluated on the basis of two written tests during the semester. The grade is given according to the following table:

Total Score (%)	Grade
0 – 49	fail (1)
50 – 59	pass (2)
60 – 69	satisfactory (3)
70 – 79	good (4)
80 – 100	excellent (5)

If a student fail to pass at first attempt, then a retake of the tests is possible.

-an offered grade:

It is not possible to obtain an offered grade in this course.

Person responsible for course: Dr. András Pongrácz, assistant professor, PhD

Lecturer: Dr. András Pongrácz, assistant professor, PhD

Title of course: Selected topics in geometry **Code**: TTMME0301	**ECTS Credit points:** 3

Type of teaching, contact hours
- lecture: 2 hours/week
- practice: -
- laboratory: -

Evaluation: exam

Workload (estimated), divided into contact hours:
- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: 32 hours
- preparation for the exam: 30 hours

Total: 90 hours

Year, semester: 1st year, 2nd semester

Its prerequisite(s):

Further courses built on it: -

Topics of course

Differentiable curves. Curvature, torsion. The fundamental theorem of curves. Surfaces in the Euclidean space. Fundamental form of surfaces. Normal curvature, principal curvatures, principal directions. Variational problem of arc-length. Geodesics. Geodesic curvature. Minimizing property of geodesics. Axioms of affine and projective planes. Projective completion of an affine plane. Duality. Vector space model of projective planes, homogenous coordinates. Perspectivities (central projections) and projectivities. Cross ratio of four points or lines, Pappus-Steiner theorem. Desargues's theorem and Pappus's theorem. Complete quadrilateral, complete quadrangle, harmonic sets of points and lines. Collineations, fundamental theorem of projective geometry. The parallel postulate, the development of hyperbolic geometry. The Cayley-Klein model of hyperbolic geometry, Poincaré disk model and upper half-plane model. Description of congruences. Spherical geometry: measuring distance on the sphere, spherical triangles. Elliptic metric.

Literature

Compulsory/Recommended Readings:

Wolfgang Kühnel: Differential Geometry: Curves – Surgaces – Manifolds, AMS, 2006.

H. S. M. Coxeter: Projective Geometry, Springer, 1974.

Patrick J. Ryan: Euclidean and non-Euclidean geometry: an analytical approach, Cambridge, 1986.

Schedule:

1st week

Regular smooth curves in Euclidean space. Reparametrization of curves. Arc length of curves. Natural parametrization, simple curves.

2nd week

Signed curvature of regular planar curves. Frenet basis. The rounding number of closed planar curves, and the theorem about it. The fundamental theorem of planar curves.

3rd week

The Frenet basis of spatial curves, Cartan matrix. Curvature and torsion, Frenet formulae.

4th week

The best approximating circle and plane of a curve at a point. The fundamental theorem of theory of curves.

5th week

Surfaces in space, some representation of surfaces. Tangent plane at a point of the surface. Normal unit vector field.

6th week

Measurement of a surface. The first main scalars of the surfaces, metrical canonical form. The length of curves on surfaces. Angle of tangential vectors. The surface area of compact surfaces.

7th week

The axioms of projective and affine planes. The projective extension of affine planes. The principle of duality.

8th week

The vector space model of projective planes, homogeneous coordinates.

9th week

Perspectivities and projective mappings. Cross ratio of 4 points and 4 concurrent lines. Pappos-Steiner theorem.

10th week

The axioms of absolute geometry. The role of the axiom of parallels, the concept of hyperbolic geometry.

11th week

The verification of the hyperbolic axioms in the case of the Cayley-Klein model. The description of congruencies.

12th week

Geometrical concepts in hyperbolic geometry: perpendicular lines, special points and lines of triangles.

13th week

Some further models of hyperbolic geometry: the circle model and the half-plane model of Poincare. Representation of isometries in these models.

14th week

Spherical geometry: distance on the sphere, theorems of spherical triangles. Elliptic metric.

Requirements:

- for a signature

Attendance at **lectures** is recommended, but not compulsory.

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. A student can’t make up any practice with another group. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence. In case of further absences, a medical certificate needs to be presented. Missed practice classes should be made up for at a later date, to be discussed with the tutor. Students are required to bring the drawing tasks and drawing instruments of the course to each practice class. Active participation is evaluated by the teacher in every class. If a student’s behavior or conduct doesn’t meet the requirements of active participation, the teacher may evaluate his/her participation as an absence because of the lack of active participation in class.

- for a grade

The course ends in an **examination**.

The minimum requirement for the mid-term and end-term tests and the examination respectively is 50%. Based on the score of the tests separately, the grade for the tests and the examination is given according to the following table:

Score	Grade
0-49	fail (1)
50-59	pass (2)
60-69	satisfactory (3)
70-79	good (4)
80-100	excellent (5)

If the score of any test is below 50, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

-an offered grade:
it may be offered for students if the average grade of the two designing tasks is at least satisfactory (3) and the average of the mid-term and end-term tests is at least satisfactory (3). The offered grade is the average of them.

Person responsible for course: Dr. László Kozma, associate professor, PhD

Lecturer: Dr. László Kozma, associate professor, PhD

Title of course: Selected topics in geometry **Code**: TTMMG0301	**ECTS Credit points:** 2

Type of teaching, contact hours
- lecture: -
- practice: 2 hours/week
- laboratory: -

Evaluation: mid-semester grade

Workload (estimated), divided into contact hours:
- lecture: -
- practice: 28 hours
- laboratory: -
- home assignment: 32 hours
- preparation for the exam: -

Total: 60 hours

Year, semester: 1st year, 2nd semester

Its prerequisite(s):

Further courses built on it: -

Topics of course

Differentiable curves. Curvature, torsion. The fundamental theorem of curves. Surfaces in the Euclidean space. Fundamental form of surfaces. Normal curvature, principal curvatures, principal directions. Variational problem of arc-length. Geodesics. Geodesic curvature. Minimizing property of geodesics. Axioms of affine and projective planes. Projective completion of an affine plane. Duality. Vector space model of projective planes, homogenous coordinates. Perspectivities (central projections) and projectivities. Cross ratio of four points or lines, Pappus-Steiner theorem. Desargues's theorem and Pappus's theorem. Complete quadrilateral, complete quadrangle, harmonic sets of points and lines. Collineations, fundamental theorem of projective geometry. The parallel postulate, the development of hyperbolic geometry. The Cayley-Klein model of hyperbolic geometry, Poincaré disk model and upper half-plane model. Description of congruences. Spherical geometry: measuring distance on the sphere, spherical triangles. Elliptic metric.

Literature

<u>Compulsory/Recommended Readings:</u>
Wolfgang Kühnel: Differential Geometry: Curves – Surgaces – Manifolds, AMS, 2006.
H. S. M. Coxeter: Projective Geometry, Springer, 1974.
Patrick J. Ryan: Euclidean and non-Euclidean geometry: an analytical approach, Cambridge, 1986.

Schedule:

1st week

Regular smooth curves in Euclidean space. Reparametrization of curves. Arc length of curves. Natural parametrization, simple curves. Examples and basic calculation.

2nd week

Signed curvature of regular planar curves. Frenet basis. The rounding number of closed planar curves, and the theorem about it. The fundamental theorem of planar curves. Examples and basic calculation.

3rd week

The Frenet basis of spatial curves, Cartan matrix. Curvature and torsion, Frenet formulae. Examples and basic calculation.

4th week

The best approximating circle and plane of a curve at a point. The fundamental theorem of theory of curves. Examples and basic calculation.

5th week

Surfaces in space, some representation of surfaces. Tangent plane at a point of the surface. Normal unit vector field. Examples and basic calculation.

6th week

Measurement of a surface. The first main scalars of the surfaces, metrical canonical form. The length of curves on surfaces. Angle of tangential vectors. The surface area of compact surfaces.

7th week

The axioms of projective and affine planes. The projective extension of affine planes. The principle of duality. Examples and basic calculation.

8th week

The vector space model of projective planes, homogeneous coordinates. Examples and basic calculation.

9th week

Perspectivities and projective mappings. Cross ratio of 4 points and 4 concurrent lines. Pappos-Steiner theorem. Examples and basic calculation.

10th week

The axioms of absolute geometry. The role of the axiom of parallels, the concept of hyperbolic geometry. Examples and basic calculation.

11th week

The verification of the hyperbolic axioms in the case of the Cayley-Klein model. The description of congruencies. Examples and basic calculation.

12th week

Geometrical concepts in hyperbolic geometry: perpendicular lines, special points and lines of triangles. Examples and basic calculation.

13th week

Some further models of hyperbolic geometry: the circle model and the half-plane model of Poincare. Representation of isometries in these models. Examples and basic calculation.

14th week

Spherical geometry: distance on the sphere, theorems of spherical triangles. Elliptic metric.

Requirements:

- for a signature

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. A student can’t make up any practice with another group. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence. In case of further absences, a medical certificate needs to be presented. Missed practice classes should be made up for at a later date, to be discussed with the tutor. Students are required to bring the drawing tasks and drawing instruments of the course to each practice class. Active participation is evaluated by the teacher in every class. If a student’s behavior or conduct doesn’t meet the requirements of active participation, the teacher may evaluate his/her participation as an absence because of the lack of active participation in class.

- for a practical grade

The minimum requirement for the mid-term and end-term tests respectively is 50%. Based on the score of the tests separately, the grade for the tests and the examination is given according to the following table:

Score	Grade
0-49	fail (1)
50-59	pass (2)
60-69	satisfactory (3)
70-79	good (4)
80-100	excellent (5)

If the score of any test is below 50, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

-an offered grade:
it may be offered for students if the average grade of the two designing tasks is at least satisfactory (3) and the average of the mid-term and end-term tests is at least satisfactory (3). The offered grade is the average of them.

Person responsible for course: Dr. László Kozma, associate professor, PhD

Lecturer: Dr. László Kozma, associate rofessor, PhD

Title of course: Operation research **Code:** TTMME0202	**ECTS Credit points:** 3

Type of teaching, contact hours

- lecture: 2 hours/week
- practice: -
- laboratory: -

Evaluation: exam

Workload (estimated), divided into contact hours:

- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 62 hours

Total: 90 hours

Year, semester: 1st year, 1st semester

Its prerequisite(s): -

Further courses built on it:-

Topics of course

Problems reducible to linear programming tasks. Extreme points of convex polyhedra, simplex algorithm and its geometry, sensitivity analysis. Duality. Transportation and assignment model, network models. Special linear programming models.

Literature

Compulsory:

-

Recommended:

- Vanderbei, R.: Linear Programming, Foundations and Extensions, Kluwer Academic Publishers, 1998.
- Bertsimas, D.; Tsitsiklis, J.: Introduction to Linear Optimization, Athena Scientific Series in Optimization and Neural Computation, 6, Athena Scientific, Belmont, 1997.

Schedule:

1st week

Introduction: The standard maximum and minimum problems, the diet problem, the optimal assignment problem

2nd week

Linear programming problems, the simplex method

3rd week

Degeneracy, lexicographic simplex method.

4th week

Effectiveness, number of steps, worst case, average case.

5th week

Duality I., special case, weak duality theorem

6th week

Duality II., strong duality theorem, dual simplex method

7th week

Matrix form, simplex tableau

8th week

Primal and dual simplex methods.

9th week

Generalized problem to standard case.

10th week

Geometry of the simplex method

11th week

The transportation problem I.

12th week

The transportation problem II.

13th week

Assignment problem I.

14th week

Assignment problem II.

Requirements:

Attendance at **lectures** is recommended, but not compulsory.

- for a grade

The course ends in an **examination**. The minimum requirement for the examination is 50%. Based on the score of the exam the grade for the examination is given according to the following table:

Score	Grade
0-49	fail (1)
50-62	pass (2)
63-76	satisfactory (3)
77-88	good (4)
89-100	excellent (5)

Person responsible for course: Dr. Fruzsina Mészáros, assistant professor, PhD

Lecturer: Dr. Fruzsina Mészáros, assistant professor, PhD

Title of course: Operation research **Code**: TTMMG0202	**ECTS Credit points:** 2

Type of teaching, contact hours
- lecture: -
- practice: 2 hours/week
- laboratory: -

Evaluation: mid-semester grade

Workload (estimated), divided into contact hours:
- lecture: -
- practice: 28 hours
- laboratory: -
- home assignment: -
- preparation for the tests: 32 hours

Total: 60 hours

Year, semester: 1st year, 1st semester

Its prerequisite(s): -

Further courses built on it:-

Topics of course

Problems reducible to linear programming tasks. Extreme points of convex polyhedra, simplex algorithm and its geometry, sensitivity analysis. Duality. Transportation and assignment model, network models. Special linear programming models.

Literature

Compulsory:

-

Recommended:

- Vanderbei, R.: Linear Programming, Foundations and Extensions, Kluwer Academic Publishers, 1998.
- Bertsimas, D.; Tsitsiklis, J.: Introduction to Linear Optimization, Athena Scientific Series in Optimization and Neural Computation, 6, Athena Scientific, Belmont, 1997.

Schedule:

1st week

Introduction: The standard maximum and minimum problems, the diet problem, the optimal assignment problem

2nd week

Linear programming problems, the simplex method

3rd week

Degeneracy, lexicographic simplex method.

4th week

Effectiveness, number of steps, worst case, average case.

5th week

Duality I., special case, weak duality theorem

6th week

Duality II., strong duality theorem, dual simplex method *7th week* Matrix form, simplex tableau *8th week* Primal and dual simplex methods. *9th week* Generalized problem to standard case. *10th week* Geometry of the simplex method *11th week* The transportation problem I. *12th week* The transportation problem II. *13th week* Assignment problem I. *14th week* Assignment problem II.
Requirements: *- for a practical* Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. A student can't make up any practice with another group. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence. In case of further absences, a medical certificate needs to be presented. Active participation is evaluated by the teacher in every class. If a student's behaviour or conduct doesn't meet the requirements of active participation, the teacher may evaluate his/her participation as an absence because of the lack of active participation in class. During the semester there are two tests: one test in the 7th week and the other test in the 14th week. The minimum requirement for the tests respectively is 50%. Based on the score of the tests, the grade for the tests is given according to the following table: Score — Grade 0-49 — fail (1) 50-62 — pass (2) 63-76 — satisfactory (3) 77-88 — good (4) 89-100 — excellent (5) If the score of any test is below 50, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.
Person responsible for course: Dr. Fruzsina Mészáros, assistant professor, PhD
Lecturer: Dr. Fruzsina Mészáros, assistant professor, PhD

Title of course: Probability theory **Code**: TTMME0401	**ECTS Credit points:** 3

Type of teaching, contact hours

- lecture: 2 hours/week
- practice: -
- laboratory: -

Evaluation: exam

Workload (estimated), divided into contact hours:

- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: 30 hours
- preparation for the exam: 32 hours

Total: 90 hours

Year, semester: 2nd year, 1st semester

Its prerequisite(s): none

Further courses built on it: -

Topics of course

Probability, random variables, distributions. Asymptotic theorems of probability theory.

Literature

Compulsory:

- A. N. Shiryayev: Probability, Springer-Verlag, Berlin, 1984.
- Ash, R. B.: Real Analysis and Probability. Academic Press, New York-London, 1972.
- Bauer, H.: Probability Theory. Walter de Gruyter, Berlin-New York. 1996.

Schedule:

1st week

Statistical observations. Numerical and graphical characteristics of the sample. Relative frequency, events, probability. Classical probability. Finite probability space.

2nd week

Kolmogorov's probability space. Properties of probability. Finite and countable probability spaces. Conditional probability, independence of events. Borel-Cantelli lemma.

3rd week

Total probability theorem, the Bayes rule. Discrete random variables. Expectation, Standard deviation. Binomial, hypergeometric, and Poisson distributions.

4th week

Random variables, distribution, cumulative distribution function. Absolutely continuous distribution, probability density function. The general notion of distribution.

5th week

Expectation, variance and median. Uniform, exponential, normal distributions.

6th week

Joint distribution function and joint probability density function of random variables. Independent random variables, correlation coefficient.

7th week

Multivariate distributions. Expectation vector and variance matrix of random a random vector. Independence of random variables.

8th week

Multivariate normal distribution, concentration ellipsoid. Sample from normal distribution. Chi-squared, Student's t, F-distributions.

9th week

Weak law of large numbers. Almost sure convergence, convergence in distribution, convergence in probability, Lp convergence.

10th week

Kolmogorov’s inequality. Kolmogorov's three-series theorem. Strong laws of large numbers.

11th week

Characteristic function and its properties. Inversion formulas. Continuity theorem

12th week

Central limit theorem. Law of the iterated logarithm. Arcsine laws.

13th week

Conditional distribution function, conditional density function, conditional expectation.

14th week

Comparison of the limit theorems.

Requirements:

- *for a grade*

he course ends in an **examination**. Based on the average of the grades of the designing tasks and the examination, the exam grade is calculated as an average of them:

- the average grade of the two designing tasks
- the result of the examination

The minimum requirement for the mid-term and end-term tests and the examination respectively is 50%. Based on the score of the tests separately, the grade for the tests and the examination is given according to the following table:

Score	Grade
0-49	fail (1)
50-69	pass (2)
70-79	satisfactory (3)
80-89	good (4)
90-100	excellent (5)

If the score of any test is below 50, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

Person responsible for course: Dr. István Fazekas, university professor, DSc

Lecturer: Dr. István Fazekas, university professor, DSc

Title of course: Probability theory **Code**: TTMMG0401	**ECTS Credit points:** 2

Type of teaching, contact hours

- lecture: -
- practice: 2 hours/week
- laboratory: -

Evaluation: exam

Workload (estimated), divided into contact hours:

- lecture: -
- practice: 28 hours
- laboratory: -
- home assignment: 16 hours
- preparation for the exam: 16 hours

Total: 60 hours

Year, semester: 2nd year, 1st semester

Its prerequisite(s): none

Further courses built on it: -

Topics of course

Probability, random variables, distributions. Asymptotic theorems of probability theory.

Literature

Compulsory:

- A. N. Shiryayev: Probability, Springer-Verlag, Berlin, 1984.
- Ash, R. B.: Real Analysis and Probability. Academic Press, New York-London, 1972.
- Bauer, H.: Probability Theory. Walter de Gruyter, Berlin-New York. 1996.

Schedule:

1st week

Statistical observations. Numerical and graphical characteristics of the sample. Relative frequency, events, probability. Classical probability. Finite probability space.

2nd week

Kolmogorov's probability space. Properties of probability. Finite and countable probability spaces. Conditional probability, independence of events. Borel-Cantelli lemma.

3rd week

Total probability theorem, the Bayes rule. Discrete random variables. Expectation, Standard deviation. Binomial, hypergeometric, and Poisson distributions.

4th week

Random variables, distribution, cumulative distribution function. Absolutely continuous distribution, probability density function. The general notion of distribution.

5th week

Expectation, variance and median. Uniform, exponential, normal distributions.

6th week

Joint distribution function and joint probability density function of random variables. Independent random variables, correlation coefficient.

7^th^ week

Multivariate distributions. Expectation vector and variance matrix of random a random vector. Independence of random variables.

8^th^ week

Multivariate normal distribution, concentration ellipsoid. Sample from normal distribution. Chi-squared, Student's t, F-distributions.

9^th^ week

Weak law of large numbers. Almost sure convergence, convergence in distribution, convergence in probability, Lp convergence.

10^th^ week

Kolmogorov's inequality. Kolmogorov's three-series theorem. Strong laws of large numbers.

11^th^ week

Characteristic function and its properties. Inversion formulas. Continuity theorem

12^th^ week

Central limit theorem. Law of the iterated logarithm. Arcsine laws.

13^th^ week

Conditional distribution function, conditional density function, conditional expectation.

14^th^ week

Comparison of the limit theorems.

Requirements:

- for a grade

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. A student can't make up any practice with another group. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence. In case of further absences, a medical certificate needs to be presented. Missed practice classes should be made up for at a later date, to be discussed with the tutor. Students are required to bring the drawing tasks and drawing instruments of the course to each practice class. Active participation is evaluated by the teacher in every class. If a student's behavior or conduct doesn't meet the requirements of active participation, the teacher may evaluate his/her participation as an absence because of the lack of active participation in class.

Students have to **submit all the two designing tasks** as scheduled minimum on a sufficient level.

During the semester there are two tests: the mid-term test in the 7th week and the end-term test in the 14th week. Students have to sit for the tests

Grades: 0-49% fail (mark 1), 50-59% satisfactory (mark 2), 60-69 % average (mark 3), 70-84% good (mark 4), 85-100 excellent (mark 5), we use rounding up (e.g. 49.3% is satisfactory).

Person responsible for course: Dr. István Fazekas, university professor, DSc

Lecturer: Dr. István Fazekas, university professor, DSc

Title of course: Graph theory and applications **Code**: TTMME0104	**ECTS Credit points:** 3

Type of teaching, contact hours

- lecture: 2 hours/week
- practice: -
- laboratory: -

Evaluation: exam

Workload (estimated), divided into contact hours:

- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 62 hours

Total: 90 hours

Year, semester: 1st year, 1st semester

Its prerequisite(s): -

Further courses built on it: TTMME0106

Topics of course

Multiply connected graphs: Menger's theorems, edge-disjoint spanning trees. Graph colourings: chromatic number, greedy vertex colouring, Brooks' theorem, Mycielski construction, perfect graphs, chromatic polynomial, chromatic index, Vizing-theorem. Independence and covering: Gallai's theorems, Kőnig's theorem, Hall's theorem, perfect mathchings in bipartite and in arbitrary graphs, augmenting path method. Extremal graph theory: Mantel's theorem, Turán's theorem. Friendship theorem, strongly regular graphs. Planar graphs, crossing number. Directed paths and cycles in directed graphs, tournaments.

Literature

Compulsory:

-

Recommended:

J. A. Bondy, U. S. R. Murty: Graph Theory, Springer, 2008.

Schedule:

1st week

Overview of fundamentals of graph theory.

2nd week

Multiply connected graphs, vertex- and edge-connectivity. Menger’s theorems, Dirac’s theorem.

3rd week

2-vertex-connected and 2-edge connected graphs. Edge disjoint spanning trees.

4th week

Chromatic number, greedy colouring, Brooks’ theorem. Mycielski construction.

5th week

Perfect graphs, examples and theorems. Chromatic polynomial, properties.

6th week

Chromatic index, Vizing’s theorem. List chromatic number, list chromatic index, total chromatic number.

7th week

Independence and coverings, Gallai’s theorems, Kőnig’s theorem.

8th week

Hall’s theorem, perfect matchings in bipartite graphs, chromatic index of bipartite graphs. Tutte’s and Petersen’s theorems on perfect matchings.

9th week

Augmenting path method for finding maximum matchings, Hungarian method. Dominating vertex sets.

10th week

Extremal graph theory, Mantel’s and Turán’s theorems.

11th week

Friendship theorem, strongly regular graphs.

12th week

Planar graphs, crossing number. Complexity of graph theoretical problems.

13th week

Directed paths and cycles in directed graphs. Gallai-Roy theorem, Stanley’s theorem.

14th week

Tournaments, Landau’s theorem, directed Hamiltonian paths and cycles in tournaments.

Requirements:

- for a signature

If the student fail the course TTMME0104, then the signature is automatically denied.

- for a grade

The course ends in oral examination. The grade is given according to the following table:

Total Score (%)	**Grade**
0 – 50	fail (1)
51 – 60	pass (2)
61 – 70	satisfactory (3)
71 – 80	good (4)
81 – 100	excellent (5)

-an offered grade:

It is not possible to obtain an offered grade in this course.

Person responsible for course: Dr. Gábor Nyul, assistant professor, PhD

Lecturer: Dr. Gábor Nyul, assistant professor, PhD

Title of course: Graph theory and applications **Code**: TTMMG0104	**ECTS Credit points:** 2

Type of teaching, contact hours
- lecture: -
- practice: 2 hours/week
- laboratory: -

Evaluation: mid-semester grade

Workload (estimated), divided into contact hours:
- lecture: -
- practice: 28 hours
- laboratory: -
- home assignment: -
- preparation for the exam: 32 hours

Total: 60 hours

Year, semester: 1st year, 1st semester

Its prerequisite(s): -

Further courses built on it: -

Topics of course

Multiply connected graphs: Menger's theorems, edge-disjoint spanning trees. Graph colourings: chromatic number, greedy vertex colouring, Brooks' theorem, Mycielski construction, perfect graphs, chromatic polynomial, chromatic index, Vizing-theorem. Independence and covering: Gallai's theorems, Kőnig's theorem, Hall's theorem, perfect mathchings in bipartite and in arbitrary graphs, augmenting path method. Extremal graph theory: Mantel's theorem, Turán's theorem. Friendship theorem, strongly regular graphs. Planar graphs, crossing number. Directed paths and cycles in directed graphs, tournaments.

Literature

Compulsory:

-

Recommended:

J. A. Bondy, U. S. R. Murty: Graph Theory, Springer, 2008.

Schedule:

1st week

Elementary exercises from graph theory.

2nd week

Vertex- and edge-connectivity of graphs.

3rd week

Chromatic number, greedy colouring.

4th week

Mycielski construction, perfect graphs.

5th week

Chromatic polynomial.

6th week

Chromatic index.

7th week

First test.

8th week

Maximum independent vertex and edge sets, minimum vertex and edge covers.

9th week

Augmenting path method, Hungarian method.

10th week

Perfect matchings.

11th week

Minimum dominating vertex sets.

12th week

Strongly regular graphs. Crossing number.

13th week

Topological ordering in directed graphs. Tournaments.

14th week

Second test.

Requirements:

- for a signature

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

- for a grade

The course is evaluated on the basis of two written tests during the semester. The grade is given according to the following table:

Total Score (%)	**Grade**
0 – 50	fail (1)
51 – 60	pass (2)
61 – 70	satisfactory (3)
71 – 80	good (4)
81 – 100	excellent (5)

If a student fail to pass at first attempt, then a retake of the tests is possible.

-an offered grade:

It is not possible to obtain an offered grade in this course.

Person responsible for course: Dr. Gábor Nyul, assistant professor, PhD

Lecturer: Dr. Gábor Nyul, assistant professor, PhD

Title of course: Algorithms in mathematics **Code**: TTMME0106	**ECTS Credit points:** 3

Type of teaching, contact hours
- lecture: 2 hours/week
- practice: -
- laboratory: -

Evaluation: exam

Workload (estimated), divided into contact hours:
- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 62 hours

Total: 90 hours

Year, semester: 1st year, 2nd semester

Its prerequisite(s): TTMME0104

Further courses built on it: -

Topics of course

Representing graphs, breadth-first search and depth-first search, finding minimal spanning trees: Kruskal's, Prim's and Boruvka's algorithms. The Bellman-Ford-algorithm. Dijkstra's algorithm. Structure of shortest paths: Floyd-Warshall-algorithm. Transitive closure of directed graphs, Johnson's algorithm on sparse graphs. Representing polynomials: discrete and fast Fourier-transformation. Number theoretical algorithms: Euclidean algorithm, operations with residue classes, Chinese remainder theorem. Computing powers. Prime tests, factorizing integers. Random prime tests, Agrawal–Kayal–Saxena prime test. Pollard's rho-algorithm.

Literature

Compulsory:

-

Recommended:

Herbert S. Wilf: Algorithms and Complexity, electronic edition, 1994.

Schedule:

1st week

Representing graphs (adjacency list and adjacency matrix representation), breadth-first search. Shortest path distance of two vertices, breadth-first trees.

2nd week

Depth-first search, predecessor subgraph, depth-first forest, timestamps. Properties of depth-first search. Classification of edges.

3rd week

Topological sort of graphs. Strongly connected component, component graph. Properties of strongly connected components.

4th week

Search for Minimum Spanning Trees, growing a Minimum Spanning Tree. The algorithms of Kruskal and Prim.

5th week

The problem of Single-Source Shortest Paths. Optimal substructure of a shortest path. Representing shortest paths (predecessor subgraph). Relaxation. Properties of shortest paths and relaxation.

6th week

The Bellman-Ford algorithm. The correctness and running time of the Bellman-Ford algorithm. The Dijkstra algorithm. The correctness and running time of the Dijkstra algorithm.

7th week

First test.

8th week

All-Pairs Shortest Paths. Shortest paths and matrix multiplication. The structure of shortest paths. The Floyd-Warshall algorithm.

9th week

Transitive closure of a directed graph. Johnson's algorithm for sparse graphs.

10th week

Sorting networks. Comparison networks. The zero-one principle. A bitonic sorting network. A merging network.

11th week

Representation of polynomials. The Discrete Fourier Transformed and the Fast Fourier Transformation algorithm. An efficient realization of the FFT.

12th week

Number Theoretical Algorithms. Euclidean algorithm, operations with residue classes, the Chinese Remainder Theorem. Fast exponentiation.

13th week

Prime-testing and prime-factorization. Probabilistic prime testing algorithms. The Agrawal–Kayal–Saxena prime test. The Pollard rho-factorization.

14th week

Second test.

Requirements:

- for a signature

If the student fail the course TTMMG0106, then the signature is automatically denied.

- for a grade

The course ends in oral examination. The grade is given according to the following table:

Total Score (%)	Grade
0 – 50	fail (1)
51 – 60	pass (2)
61 – 70	satisfactory (3)
71 – 85	good (4)
86 – 100	excellent (5)

-an offered grade:

It is possible to obtain an offered grade on the basis of two written test during the semester. The grade is given according to the following table:

Total Score (%)	Grade
0 – 50	fail (1)
51 – 60	pass (2)

61 – 70	satisfactory (3)
71 – 85	good (4)
86 – 100	excellent (5)

Person responsible for course: Prof. Dr. Attila Bérczes, university professor, DSc

Lecturer: Prof. Dr. Attila Bérczes, university professor, DSc

Title of course: Algorithms in mathematics **Code**: TTMMG0106	**ECTS Credit points:** 2

Type of teaching, contact hours
- lecture: -
- practice: 2 hours/week
- laboratory: -

Evaluation: mid-semester grade

Workload (estimated), divided into contact hours:
- lecture: -
- practice: 28 hours
- laboratory: -
- home assignment: -
- preparation for the exam: 32 hours

Total: 60 hours

Year, semester: 1st year, 2nd semester

Its prerequisite(s): TTMME0104

Further courses built on it: -

Topics of course

Representing graphs, breadth-first search and depth-first search, finding minimal spanning trees: Kruskal's, Prim's and Boruvka's algorithms. The Bellman-Ford-algorithm. Dijkstra's algorithm. Structure of shortest paths: Floyd-Warshall-algorithm. Transitive closure of directed graphs, Johnson's algorithm on sparse graphs. Representing polynomials: discrete and fast Fourier-transformation. Number theoretical algorithms: Euclidean algorithm, operations with residue classes, Chinese remainder theorem. Computing powers. Prime tests, factorizing integers. Random prime tests, Agrawal–Kayal–Saxena prime test. Pollard's rho-algorithm.

Literature

Compulsory:

-

Recommended:

Herbert S. Wilf: Algorithms and Complexity, electronic edition, 1994.

Schedule:

1st week

Representation of graphs in computer algebra systems. Programming the breadth-first search.

2nd week

Programming the depth-first search.

3rd week

Programming the Kruskal algorithm.

4th week

Programming the Prim algorithm.

5th week

Programming the Bellmann-Ford algorithm.

6th week

Programming the Dijkstra algorithm.

7th week

Programming the Floyd-Warshall algorithm.

8th week

Programming the Johnson algorithm.

9th week

Programming sorting networks.

10th week

Programming the Fast Fourier Transform algorithm.

11th week

Programming the Euclidean algorithm and the fast exponentiation.

12th week

Programming the Miller-Rabin test.

13th week

Programming the Pollard rho-factorization.

14th week

Programming the Agrawal–Kayal–Saxena prime test.

Requirements:

- for a signature

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

- for a grade

The course is evaluated on the basis of two written tests during the semester. The grade is given according to the following table:

Total Score (%)	**Grade**
0 – 50	fail (1)
51 – 60	pass (2)
61 – 70	satisfactory (3)
71 – 85	good (4)
86 – 100	excellent (5)

If a student fail to pass at first attempt, then a retake of the test is possible.

-an offered grade:

It is not possible to obtain an offered grade in this course.

Person responsible for course: Prof. Dr. Attila Bérczes, university professor, DSc

Lecturer: Prof. Dr. Attila Bérczes, university professor, DSc

Title of course: Convex optimization **Code**: TTMME0205	**ECTS Credit points:** 3

Type of teaching, contact hours
- lecture: 2 hours/week
- practice:
- laboratory:

Evaluation: exam

Workload (estimated), divided into contact hours:
- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 62 hours

Total: 90 hours

Year, semester:

Its prerequisite(s): TTMMG0205

Further courses built on it: -

Topics of course

Hull operations and their representations. The Stone–Kakutani separation theorem. Algebraic interior and algebraic closure. The intersection of the algebraic closure of complementary convex sets; separation of convex sets by linear functions. The Dubovickij–Miljutin theorem and its consequences. The Bernstein–Doetsch theorem for linear functions; the topological form of the separation theorems. Convex and sublinear functions; the maximum theorem and its consequences. Subgradient and directional derivative of convex functions. Rules of calculus. The Bernstein–Doetsch theorem for convex functions. Distance function, tangent cone, normal cone. The minimum of convex conditional extremum problems; primal and dual conditions. The convex Fermat principle. Penalty function. The Karush–Kuhn–Tucker theorem and its consequence. Slater condition and Slater theorem.

Literature

Compulsory:

T. R. Rockafellar: Convex Analysis, Princeton University Press, Princenton, N. J., 1970.

J. M. Borwein and A. S. Lewis: Convex Analysis and Nonlinear Optimization, CMS Books in Mathematics, Springer, New York, 2006.

Recommended: -

Schedule:

1st week

Hull operations and their representations. The Stone–Kakutani separation theorem.

2nd week

Algebraic interior and algebraic closure. The intersection of the algebraic closure of complementary convex sets.

3rd week

Separation of convex sets by linear functions.

4th week

The Dubovitsky–Milyutin theorem and its consequences.

5th week
The Bernstein–Doetsch theorem for linear functions.

6th week
The topological form of the separation theorems.

7th week
Convex and sublinear functions.

8th week
The maximum theorem and its consequences.

9th week
Subgradient and directional derivative of convex functions.

10th week
The Bernstein–Doetsch theorem for convex functions.

11th week
Distance function, tangent cone, normal cone.

12th week
The minimum of convex conditional extremum problems; primal and dual conditions.

13th week
The convex Fermat principle. Penalty function. The Karush–Kuhn–Tucker theorem and its consequence.

14th week
Slater condition and Slater theorem.

Requirements:

The course ends in an oral or written examination. Two assay questions are chosen randomly from the list of assays. In case one of them is incomplete, the examination ends with a fail. In lack of the knowledge of proofs, at most satisfactory can be achieved. The grade for the examination is given according to the following table:

Score	Grade
0-59%	fail (1)
60-69%	pass (2)
70-79%	satisfactory (3)
80-89%	good (4)
90-100%	excellent (5)

In general, the EDUCATION AND EXAMINATION RULES AND REGULATIONS have to be taken into account.

Person responsible for course: Dr. Mihály Bessenyei, associate professor, PhD

Lecturer: Dr. Mihály Bessenyei, associate professor, PhD

Title of course: Convex optimization **Code**: TTMMG0205	**ECTS Credit points: 2**

Type of teaching, contact hours
- lecture:
- practice: 2 hours/week
- laboratory:

Evaluation: mid-semester grade

Workload (estimated), divided into contact hours:
- lecture: -
- practice: 28 hours
- laboratory: -
- home assignment: 32 hours
- preparation for the exam: -

Total: 60 hours

Year, semester: odd semesters

Its prerequisite(s): -

Further courses built on it: -

Topics of course

Hull operations and their representations. The Stone–Kakutani separation theorem. Algebraic interior and algebraic closure. The intersection of the algebraic closure of complementary convex sets; separation of convex sets by linear functions. The Dubovickij–Miljutin theorem and its consequences. The Bernstein–Doetsch theorem for linear functions; the topological form of the separation theorems. Convex and sublinear functions; the maximum theorem and its consequences. Subgradient and directional derivative of convex functions. Rules of calculus. The Bernstein–Doetsch theorem for convex functions. Distance function, tangent cone, normal cone. The minimum of convex conditional extremum problems; primal and dual conditions. The convex Fermat principle. Penalty function. The Karush–Kuhn–Tucker theorem and its consequence. Slater condition and Slater theorem.

Literature

Compulsory:

T. R. Rockafellar: Convex Analysis, Princeton University Press, Princenton, N. J., 1970.

J. M. Borwein and A. S. Lewis: Convex Analysis and Nonlinear Optimization, CMS Books in Mathematics, Springer, New York, 2006.

Recommended: -

Schedule:

1st week

Linear subspaces, affine subspaces, convex cones, convex subsets in linear spaces.

2nd week

Linear and sublinear functions, affine functions and convex functions.

3rd week

Linear hull, affine hull, cone hull and convex hull in finite dimension. The drop theorem.

4th week

Linear hull, affine hull, cone hull and convex hull in infinite dimension.

5th week

Polyhedrons and polytopes in finite dimension.

6th week

Algebraic interior, algebraic open sets. Convex sets in topological vector spaces.

7th week

Mid-term test.

8th week

Separation of convex sets with linear mapping.

9th week

Directional derivative of convex functions. Calculus with respect to convex cones. The maximum function.

10th week

Subgradients of convex functions.

11th week

Extrema via Lagrange multipliers.

12th week

Applications of the Karush–Kuhn–Tucker theorem.

13th week

Applications of the Karush–Kuhn–Tucker theorem.

14th week

End-term test.

Requirements:

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. A student can't make up any practice with another group. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence. In case of further absences, a medical certificate needs to be presented. Missed practice classes should be made up for at a later date, to be discussed with the tutor.

The course finishes with a grade, which is based on the total sum of points of the mid-term test (in the 7th week) and the end-term test (in the 14th week). One of the test can be repeated. The final grade is given according to the following table:

Score	Grade
0-59%	fail (1)
60-69%	pass (2)
70-79%	satisfactory (3)
80-89%	good (4)
90-100%	excellent (5)

In general, the EDUCATION AND EXAMINATION RULES AND REGULATIONS have to be taken into account.

Person responsible for course: Dr. Mihály Bessenyei, associate professor, PhD

Lecturer: Dr. Mihály Bessenyei, associate professor, PhD

Title of course: Discrete optimization **Code**: TTMME0107	**ECTS Credit points:** 3

Type of teaching, contact hours
- lecture: 2 hours/week
- practice: -
- laboratory: -

Evaluation: exam

Workload (estimated), divided into contact hours:
- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 62 hours

Total: 90 hours

Year, semester: 1st year, 2nd semester

Its prerequisite(s):

Further courses built on it:

Topics of course

Theoretical background of discrete optimization problems. Totally unimodular matrices, integer linear programming, Hoffman-Kruskal theorem. Assignment problem, quadratic assignment problem, set covering problem, Chinese postman problem, travelling salesman problem, Steiner-tree problem, bin packing problem. Max flow−min cut problem, Ford-Fulkerson theorem, Edmonds-Karp theorem. Greedy algorithm for downward closed family of sets, matroids.

Literature

Compulsory:

-

Recommended:

Bernhard Korte, Jens Vygen: Combinatorial Optimization, Springer-Verlag, 2006.
Dieter Jungnickel: Graphs, Networks and Algorithms, Springer-Verlag, 2008.
Vijay V. Vazirani: Approximation Algorithms, Springer-Verlag, 2001.

Schedule:

1st week

Theoretical background of discrete optimization problems, general methods: exhaustive search, branch and bound method, suboptimal algorithms.

2nd week

Totally unimodular matrices, elementary properties, equivalents, examples (incidence matrices of directed and bipartite graphs, interval matrices), Heller's theorem.

3rd week

Linear programming, integer linear programming, Hoffman-Kruskal theorem. Graph theoretical problems using integer linear programming (independent vertex and edge sets, vertex and edge cover).

4th week

Assignment problem, Hungarian method. Quadratic assignment problem.

5th week

Unweighted and weighted vertex cover problem, suboptimal algorithms.

6th week

Set cover problem, Chvátal's method.

7th week

Chinese postman problem, method.

8th week

Travelling salesman problem, metric and nonmetric variants, suboptimal methods in the metric case, Christofides' method.

9th week

Steiner tree problem, suboptimal method.

10th week

Bin packing problem, NF, FF, FFD methods.

11th week

Networks and flows, maximum flow−minimum cut problem, Ford-Fulkerson theorem.

12th week

Ford-Fulkerson method, integer capacities, Edmonds-Karp theorem. Maximum flow–minimum cut problems and linear programming.

13th week

Networks with multiple sources and sinks, networks with maximal capacity. The Ford-Fulkerson theorem and its theoretical consequences.

14th week

Greedy algorithm for downward closed set systems, matroids, examples.

Requirements:

- for a signature

If the student fail the course TTMMG0107, then the signature is automatically denied.

- for a grade

The course ends in oral examination. The grade is given according to the following table:

Total Score (%)	Grade
0 – 50	fail (1)
51 – 60	pass (2)
61 – 70	satisfactory (3)
71 – 80	good (4)
81 – 100	excellent (5)

-an offered grade:

It is not possible to obtain an offered grade in this course.

Person responsible for course: Dr. Gábor Nyul, assistant professor, PhD

Lecturer: Dr. Gábor Nyul, assistant professor, PhD

Title of course: Discrete optimization Code: TTMMG0107	ECTS Credit points: 2

Type of teaching, contact hours

- lecture: -
- practice: 2 hours/week
- laboratory: -

Evaluation: mid-semester grade

Workload (estimated), divided into contact hours:

- lecture: -
- practice: 28 hours
- laboratory: -
- home assignment: -
- preparation for the exam: 32 hours

Total: 60 hours

Year, semester: 2nd year, 2nd semester

Its prerequisite(s): -

Further courses built on it: -

Topics of course

Theoretical background of discrete optimization problems. Totally unimodular matrices, integer linear programming, Hoffman-Kruskal theorem. Assignment problem, quadratic assignment problem, set covering problem, Chinese postman problem, travelling salesman problem, Steiner-tree problem, bin packing problem. Max flow−min cut problem, Ford-Fulkerson theorem, Edmonds-Karp theorem. Greedy algorithm for downward closed family of sets, matroids.

Literature

Compulsory:

-

Recommended:

Bernhard Korte, Jens Vygen: Combinatorial Optimization, Springer-Verlag, 2006.
Dieter Jungnickel: Graphs, Networks and Algorithms, Springer-Verlag, 2008.
Vijay V. Vazirani: Approximation Algorithms, Springer-Verlag, 2001.

Schedule:

1st week

Basic graph algorithms.

2nd week

PERT method, critical paths.

3rd week

Totally unimodular matrices.

4th week

Linear programming. Rearrangement theorem.

5th week

Assignment problem.

6th week

Set cover problem.

7th week

First test.

8th week

Chinese postman problem.

9th week

Travelling salesman problem.

10th week

Steiner tree problem. Bin packing problem.

11th week

Networks and flows.

12th week

Maximum flow−minimum cut problem, Ford-Fulkerson method.

13th week

Generalized networks.

14th week

Second test.

Requirements:

- for a signature

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

- for a grade

The course is evaluated on the basis of two written tests during the semester. The grade is given according to the following table:

Total Score (%)	Grade
0 – 50	fail (1)
51 – 60	pass (2)
61 – 70	satisfactory (3)
71 – 80	good (4)
81 – 100	excellent (5)

If a student fail to pass at first attempt, then a retake of the tests is possible.

-an offered grade:

It is not possible to obtain an offered grade in this course.

Person responsible for course: Dr. Gábor Nyul, assistant professor, PhD

Lecturer: Dr. Gábor Nyul, assistant professor, PhD

Title of course: Application of ordinary differential equations **Code**: TTMME0207	**ECTS Credit points:** 3
Type of teaching, contact hours - lecture: 2 hours/week - practice: - - laboratory: -	
Evaluation: exam	
Workload (estimated), divided into contact hours: - lecture: 28 hours - practice: - laboratory: - - home assignment: - - preparation for the exam: 62 hours Total: 90 hours	
Year, semester: 2nd year, 1nd semester	
Its prerequisite(s): -	
Further courses built on it: -	

Topics of course

Autonomous systems of differential equations and their phase spaces. Stability of differencial equations, the theorems of Lyapunov, the direct method of Lyapunov. Boundary value problems and eigenvalue problems. Green function. Existence and uniqueness theorems. Maximum and minimum principles. Nonlinear boundary value problems. Sturm-Liouville eigenvalue problems. Rotationally symmetric elliptic problems. Diffeomorphisms and their symmetries. The application of the one-parameter symmetry group to integration of equations. Calculus of variations, the Euler–Lagrange differential equations, the invariance of the Euler–Lagrange differential equations, the canonical form of the Euler–Lagrange differential equations, the first integrals of the Euler–Lagrange differential equations. The Noether theorem. Principle of the least action.

Literature

Compulsory: –

Recommended:

[1] **V. I. Arnol'd**, Ordinary differential equations, Translated from the Russian by Roger Cooke. Second printing of the 1992 edition. Universitext. *Springer-Verlag, Berlin*, 2006. ii+334 pp. ISBN: 978-3-540-34563-3;

[2] **V. I. Arnol'd**, Mathematical methods of classical mechanics. Translated from the 1974 Russian original by K. Vogtmann and A. Weinstein. Corrected reprint of the second (1989) edition. Graduate Texts in Mathematics, 60. *Springer-Verlag, New York*, 1989. xvi+516 pp. ISBN: 0-387-96890-3

[3] **V. I. Arnol'd**, Geometrical methods in the theory of ordinary differential equations. Translated from the Russian by Joseph Szücs [József M. Szűcs]. Second edition. Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], 250. *Springer-Verlag, New York*, 1988. xiv+351 pp. ISBN: 0-387-96649-8

[4] **B. Dacorogna**, Introduction to the calculus of variations, 2nd ed., London: Imperial College Press, 2008.

[5] **A. D. Ioffe, V. M. Tihomirov**, Theory of extremal problems, Studies in Mathematics and its Applications, 6. North-Holland Publishing Co., Amsterdam-New York, 1979.

[6] **W. Walter**, Gewöhnliche Differentialgleichungen – Eine Einfürung, 7. Auflage, Springer, 2000.

Schedule:

1st week

Autonomous differential equations and their phase spaces: Autonomous systems of ordinary differential equations, phase spaces, equilibrium point, cycles.

2nd week

Stability theory of ordinary differential equations, Theorems of Lyapunov.

3rd week

The direct method of Lyapunov.

4th week

Boundary value problems and eigenvalue problems, Sturm eigenvalue problems, fundamental solutions, the Green function.

5th week

Non-linear boundary value problems, minimum and maximum principles.

6th week

Sturm–Liouville eigenvalue problems, spherical symmetric solutions of elliptic partial differential equations.

7th week

One-parameter transformations groups, one-parameter diffeomorphism groups.

8th week

Actions of diffeomorphisms on vector fields, symmetries, the Rectification Theorem.

9th week

Variations of functionals, bilinear and quadratic functionals, second order variations of functionals.

10th week

Extrema of functionals, the Euler–Lagrange equations.

11th week

Invariance of the Euler–Lagrange differential equations, canonical form of the Euler–Lagrange differential equations, first integrals of the Euler–Lagrange differential equations.

12th week

The Theorem of Noether, the Principle of the least action.

13th week

Sufficient conditions for the extrema, solutions to the most classical problems in the theory of calculus of variations.

14th week

Calculus of variations for multivariate functions, derivation of the most classical second order partial differential equations (Laplace equation, wave equation, heat equation, minimal surface equation)

Requirements:

Attendance at **lectures** is recommended, but not compulsory.

The course ends in an oral **examination**.

Person responsible for course: Dr. Eszter Novák-Gselmann, assistant professor, PhD

Lecturer: Dr. Eszter Novák-Gselmann, assistant professor, PhD

Title of course: Application of ordinary differential equations **Code**: TTMMG0207 (practice)	**ECTS Credit points:** 2
Type of teaching, contact hours - lecture: - - practice: 2 hours/week - laboratory: -	
Evaluation: mid-semester grade	
Workload (estimated), divided into contact hours: - lecture: - practice: 28 hours - laboratory: - - home assignment: 32 hours - preparation for the exam: - Total: 60 hours	
Year, semester: 2nd year, 1nd semester	
Its prerequisite(s): -	
Further courses built on it: -	

Topics of course

Autonomous systems of differential equations and their phase spaces. Stability of differencial equations, the theorems of Lyapunov, the direct method of Lyapunov. Boundary value problems and eigenvalue problems. Green function. Existence and uniqueness theorems. Maximum and minimum principles. Nonlinear boundary value problems. Sturm-Liouville eigenvalue problems. Rotationally symmetric elliptic problems. Diffeomorphisms and their symmetries. The application of the one-parameter symmetry group to integration of equations. Calculus of variations, the Euler–Lagrange differential equations, the invariance of the Euler–Lagrange differential equations, the canonical form of the Euler–Lagrange differential equations, the first integrals of the Euler–Lagrange differential equations. The Noether theorem. Principle of the least action.

Literature

Compulsory: –

Recommended:

[1] **V. I. Arnol'd**, Ordinary differential equations, Translated from the Russian by Roger Cooke. Second printing of the 1992 edition. Universitext. *Springer-Verlag, Berlin*, 2006. ii+334 pp. ISBN: 978-3-540-34563-3;

[2] **V. I. Arnol'd**, Mathematical methods of classical mechanics. Translated from the 1974 Russian original by K. Vogtmann and A. Weinstein. Corrected reprint of the second (1989) edition. Graduate Texts in Mathematics, 60. *Springer-Verlag, New York*, 1989. xvi+516 pp. ISBN: 0-387-96890-3

[3] **V. I. Arnol'd**, Geometrical methods in the theory of ordinary differential equations. Translated from the Russian by Joseph Szücs [József M. Szűcs]. Second edition. Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], 250. *Springer-Verlag, New York*, 1988. xiv+351 pp. ISBN: 0-387-96649-8

[4] **B. Dacorogna**, Introduction to the calculus of variations, 2nd ed., London: Imperial College Press, 2008.

[5] **A. D. Ioffe, V. M. Tihomirov**, Theory of extremal problems, Studies in Mathematics and its Applications, 6. North-Holland Publishing Co., Amsterdam-New York, 1979.

[6] **W. Walter**, Gewöhnliche Differentialgleichungen – Eine Einfürung, 7. Auflage, Springer, 2000.

Schedule:

1st week

Autonomous differential equations and their phase spaces: Autonomous systems of ordinary differential equations, phase spaces, equilibrium point, cycles.

2nd week

Stability theory of ordinary differential equations, Theorems of Lyapunov.

3rd week

The direct method of Lyapunov.

4th week

Boundary value problems and eigenvalue problems, Sturm eigenvalue problems, fundamental solutions, the Green function.

5th week

Non-linear boundary value problems, minimum and maximum principles.

6th week

Sturm–Liouville eigenvalue problems, spherical symmetric solutions of elliptic partial differential equations.

7th week

One-parameter transformations groups, one-parameter diffeomorphism groups, Actions of diffeomorphisms on vector fields, symmetries, the Rectification Theorem.

8th week

Variations of functionals, bilinear and quadratic functionals, second order variations of functionals.

9th week

Extrema of functionals, the Euler–Lagrange equations.

10th week

Invariance of the Euler–Lagrange differential equations, canonical form of the Euler–Lagrange differential equations, first integrals of the Euler–Lagrange differential equations.

11th week

The Theorem of Noether, the Principle of the least action.

12th week

Sufficient conditions for the extrema, solutions to the most classical problems in the theory of calculus of variations.

13th week

Calculus of variations for multivariate functions, derivation of the most classical second order partial differential equations (Laplace equation, wave equation, heat equation, minimal surface equation)

14th week

Test writing

Requirements:

Participation at practice classes is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. A student can’t make up any practice with another group. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence. In case of further absences, a medical certificate needs to be presented. Missed practice classes should be made up for at a later date, to be discussed with the tutor. Active participation is evaluated by the teacher in every class. If a student’s behaviour does not meet the requirements of active participation, the teacher may evaluate his/her participation as an absence because of the lack of active participation in class.

During the semester there is one written test, in the 14th week.

The minimum requirement for the test is 66%. The grade for the tests is given according to the following table:

Score	Grade
0-65	fail (1)
66-69	pass (2)
70-80	satisfactory (3)
81-90	good (4)
91-100	excellent (5)

If the score of the test is below 66%, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

Person responsible for course: Dr. Eszter Novák-Gselmann, assistant professor, PhD

Lecturer: Dr. Eszter Novák-Gselmann, assistant professor, PhD.

Title of course: Partial differential equations **Code**: TTMME0204	**ECTS Credit points:** 3
Type of teaching, contact hours - lecture: 2 hours/week - practice: - - laboratory: -	
Evaluation: exam	
Workload (estimated), divided into contact hours: - lecture: 28 hours - practice: - - laboratory: - - home assignment: - - preparation for the exam: 62 hours Total: 90 hours	
Year, semester: 2nd year, 2nd semester	
Its prerequisite(s):	
Further courses built on it: -	

Topics of course

Examples in physics. First order equations: homogeneous linear equations, quasilinear equations and Cauchy problem for general equations. Higher order equations, the Cauchy–Kovalevskaya theorem. Classification and canonical form of second order equations. One, two and three dimensional wave equation. Inhomogeneous wave equation. The Laplace and Poisson equation, Green functions, harmonic functions, maximum principle. Boundary value problem for the Laplace and Poisson equations. The heat equation. Sobolev spaces, weak solutions.

Literature

Compulsory: -
Recommended:
- V. I. Arnold: Lectures on Partial Differential Equations, Springer, Berlin, 2004.

Schedule:

1st week Introduction. Examples in physics. Main types of partial differential equations.

2nd week First order equations: first integrals of ordinary differential equations, homogeneous linear first order partial differential equations

3rd week First order quasilinear equations and Cauchy problems for general first order equations.

4th week Higher order equations, the Cauchy–Kovalevskaya theorem. Classification of second order equations.

5th week Canonical form of second order linear equations with constant coefficients.

6th week Canonical form of two dimensional second order semilinear equations.

7th week One dimensional wave equation: the equation on the real line. Initial value problem on the real line, initial and boundary value problem on bounded intervals.

8th week Initial value problem for the two and three dimensional wave equation. Inhomogeneous wave equation.

9th week Basic solutions of the Poisson equation. Green functions.

10th week Poisson formula, harmonic functions, maximum principle, monotonicity principle.

11th week Boundary value problem for the Laplace and Poisson equations.

12th week Heat kernel, initial value problem for the heat equation.

13th week Weak derivative, Sobolev spaces, Friedrichs-inequality, equivalent norms.

14th week Weak solutions of the Poisson equation, the Lax-Milgram lemma.

Requirements:

- for a grade

The course ends in an **examination**. The minimum requirement for the examination is 50%. Based on the score of the exam the grade for the examination is given according to the following table

Score	Grade
0-49	fail (1)
50-59	pass (2)
60-74	satisfactory (3)
75-89	good (4)
90-100	excellent (5)

Person responsible for course: Dr. Borbála Fazekas, assistant professor, PhD

Lecturer: Dr. Borbála Fazekas, assistant professor, PhD

Title of course: Partial differential equations **Code**: TTMMG0204	**ECTS Credit points:** 2
Type of teaching, contact hours - lecture: - - practice: 2 hours/week - laboratory: -	
Evaluation: mid-semester grade	
Workload (estimated), divided into contact hours: - lecture: - - practice: 28 hours - laboratory: - - home assignment: - - preparation for the exam: 32 hours Total: 60 hours	
Year, semester: 2nd year, 2nd semester	
Its prerequisite(s):	
Further courses built on it: -	

Topics of course

Examples in physics. First order equations: homogeneous linear equations, quasilinear equations and Cauchy problems for general equations. Higher order equations, the Cauchy–Kovalevskaya theorem. Classification and canonical form of second order equations. One, two and three dimensional wave equation. Inhomogeneous wave equation. The Laplace and Poisson equation, Green functions, harmonic functions, maximum principle. Boundary value problem for the Laplace and Poisson equations. The heat equation. Sobolev spaces, weak solutions.

Literature

Compulsory: -

Recommended:

- V. I. Arnold: Lectures on Partial Differential Equations, Springer, Berlin, 2004.

Schedule:

1st week Introduction. Examples in physics. Main types of partial differential equations.

2nd week First order equations: first integrals of ordinary differential equations, homogeneous linear first order partial differential equations.

3rd week First order quasilinear equations and Cauchy problem for general first order equations.

4th week Higher order equations, the Cauchy–Kovalevskaya theorem. Classification of second order equations.

5th week Canonical form of second order linear equations with constant coefficients.

6th week Canonical form of two dimensional second order semilinear equations.

7th week One dimensional wave equation: the equation on the real line. Initial value problem on the real line, initial and boundary value problém on bounded intervals.

8th week Initial value problem for the two and three dimensional wave equation. Inhomogeneous wave equation.

9th week Basic solutions of the Poisson equation. Green functions.

10th week Poisson formula, harmonic functions, maximum principle, monotonicity principle.

11th week Boundary value problem for the Laplace and Poisson equations.

12th week Heat kernel, initial value problem for the heat equation.

13th week Weak derivative, Sobolev spaces, Friedrichs-inequality, equivalent norms. Weak solutions of the Poisson equation, the Lax-Milgram lemma.

14th week Test

Requirements:

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. A student can't make up any practice with another group. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence. In case of further absences, a medical certificate needs to be presented. Active participation is evaluated by the teacher in every class. If a student's behavior or conduct doesn't meet the requirements of active participation, the teacher may evaluate his/her participation as an absence because of the lack of active participation in class.

During the semester there are two tests: one test in the 7th week and the other test in the 14th week. The minimum requirement for the tests respectively is 50%. Based on the score of the tests, the grade for the tests is given according to the following table:

Score	Grade
0-49	fail (1)
50-59	pass (2)
60-74	satisfactory (3)
75-89	good (4)
90-100	excellent (5)

If the score of any test is below 50, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

Person responsible for course: Dr. Borbála Fazekas, assistant professor, PhD

Lecturer: Dr. Borbála Fazekas, assistant professor, PhD

Title of course: Stochastic processes **Code**: TTMME0402	**ECTS Credit points:** 3

Type of teaching, contact hours
- lecture: 2 hours/week
- practice: -
- laboratory: -

Evaluation: exam

Workload (estimated), divided into contact hours:
- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 62 hours

Total: 90 hours

Year, semester: 1st year, 2nd semester

Its prerequisite(s): none

Further courses built on it: -

Topics of course

General notion of conditional expected value, discrete and continuous time Markov chains, discrete time martingales, Wiener processes, stochastic integration with Wiener process (Itô integral), Itô's formula, stochastic differential equations, diffusion processes.

Literature

Compulsory:
- I. Karatzas, S. E. Shreve: Brownian Motion and Stochastic Calculus, Springer-Verlag, 1991.
- N. Shiryayev: Probability, 2nd edition, Springer-Verlag, 1995.

Recommended:
- S. M. Ross: Introduction to Probability Models, 10th edition, Academic Press, 2009.

Schedule:

1st week

Conditional expected value with respect to sigma algebra: definition, existence, Jensen-inequality, tower rule, Fatou-lemma, monotone dominated convergence theorem.

2nd week

Definition of stochastic processes, independent increments, stationary increments, finite dimensional distributions of a stochastic process, expected value function, covariance function, cylender sets, Kolmogorov existence theorem.

3rd week

Discrete time Markov-chain: definition, existence theorem of Markov-chains, initial distribution, transition probability matrix, Kolmogorov-Chapman equations.

4th week

Simulation of Markov-chains knowing the initial distributions and transition probabilities, classification of states of a Markov-chain.

5th week

Discrete time Markov-chain: accessibility, essential states, inessential states, closeness, irreducibility, periodicity, recurrence, criteria of recurrence, stacionarity, ergodicity, convergence of transition probabilities

6th week

Discrete time martingales: definition, the basic probabilities, Doob's decomposition theorem, stopping time, optional stopping theorem.

7th week

Discrete time martingales: Wald-identity, Doob's martingale maximal inequalities, convergence of martingales and submartingales.

8th week

Continuous time Markov-chains: transition probabilities functions, Kolmogorov-Chapman equalities, standardization, infinitesimal generators/matrices and its interpretation, conservation, system of backward and forward Kolmogorov differential equations.

9th week

Continuous time Markov-chains: recurrence, asymptotic behaviour of transition probabilities, ergodic and null-states, stationary distribution, birth and death processes, Karlin-McGregor-theorem.

10th week

The existence of standard Wiener-processes, Kolmogorov continuity theorem, the basic properties of Wiener-processes, transition probability density function.

11th week

Definition and basic properties of Gaussian processes; Wiener-processes, as a special case of Gaussian processes, the hitting time, examination of bounded variation and differentiation.

12th week

Definition and basic properties of stochastic integral with respect to Wiener processes (Itô-integral).

13th week

Itô's formula and its applications to determine stochastic integrals.

14th week

Stochastic differential equations: strong and weak solutions; diffusion processes, examples (principally of the area of financial mathematics). Kolmogorov-equations.

Requirements:

The course ends in an **examination**. Based on the average of the grades of the designing tasks and the examination, the exam grade is calculated as an average of them:

- the average grade of the two designing tasks
- the result of the examination

The minimum requirement for the mid-term and end-term tests and the examination respectively is 50%. Based on the score of the tests separately, the grade for the tests and the examination is given according to the following table:

Score	Grade
0-49	fail (1)
50-69	pass (2)
70-79	satisfactory (3)
80-89	good (4)
90-100	excellent (5)

If the score of any test is below 50, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.
Person responsible for course: Dr. Patricia Szokol, assistant professor, PhD
Lecturer: Prof. Dr. István Fazekas, university professor, DSc Dr. Patricia Szokol, associate professor, PhD

Title of course: Stochastic processes **Code**: TTMMG0402	**ECTS Credit points:** 2

Type of teaching, contact hours
- lecture: -
- practice: 2 hours/week
- laboratory: -

Evaluation: mid-semester grade

Workload (estimated), divided into contact hours:
- lecture: -
- practice: 28 hours
- laboratory: -
- home assignment: 32 hours
- preparation for the exam:

Total: 60 hours

Year, semester: 1st year, 2nd semester

Its prerequisite(s): none

Further courses built on it: -

Topics of course

General notion of conditional expected value, discrete and continuous time Markov chains, discrete time martingales, Wiener processes, stochastic integration with the Wiener process (Itô integral), Itô's formula, stochastic differential equations, diffusion processes.

Literature

Compulsory:
- I. Karatzas, S. E. Shreve: Brownian Motion and Stochastic Calculus, Springer-Verlag, 1991.
- N. Shiryayev: Probability, 2nd edition, Springer-Verlag, 1995.

Recommended:
- S. M. Ross: Introduction to Probability Models, 10th edition, Academic Press, 2009.

Schedule:

1st week

Conditional expected value with respect to sigma algebra: examples to practice the definition, and the basic properties.

2nd week

Examples for stochastic processes; exercises to practice the notion of independent increments, stationary increments, finite dimensional distributions of a stochastic process; exercises to calculate expected value function and covariance function.

3rd week

Discrete time Markov-chains: examples and exercises to understand the definition and to practice initial distribution, transition probability matrix, Kolmogorov-Chapman equations.

4th week

Discrete time Markov-chains: exercises to practice the classification of states of Markov-chain. Simulation of Markov-chains using the statistical software R.

5th week

Discrete time Markov-chains: exercises to apply the criteria of recurrence, to determine the stationary distribution and to examine the ergodicity and the convergence of transition probabilities.

6th week

Discrete time martingales: exercises to practice the definition, basic probabilities and optional stopping theorem.

7th week

Discrete time martingales: exercises to practice the Wald-identity, the convergence of martingales and submartingales.

8th week

Continuous time Markov-chains: examples for infinitezimal generators and exercises to apply the system of backward and forward Kolmogorov differential equations.

9th week

Continuous time Markov-chains: exercises for the examination of the recurrance, asymptotic behaviour of transition probabilities, to practice the notion of the ergodic and null-states and to determine stationary distributions.

10th week

Exercises and examples for Wiener processes.

11th week

Examples and exercises for Gaussian processes and for hitting time of Wiener processes.

12th week

Examples and exercises for stochastic integral with respect to Wiener processes (Itô-integral). Itô's formula and its applications to determine stochastic integrals.

13th week

Examples and exercises for stochastic differential equations and for diffusion processes.

14th week

End-term test.

Requirements:

- *for a grade*

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. A student can't make up any practice with another group. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence. In case of further absences, a medical certificate needs to be presented. Missed practice classes should be made up for at a later date, to be discussed with the tutor. Students are required to bring the drawing tasks and drawing instruments of the course to each practice class. Active participation is evaluated by the teacher in every class. If a student's behavior or conduct doesn't meet the requirements of active participation, the teacher may evaluate his/her participation as an absence because of the lack of active participation in class.

Students have to **submit all the two designing tasks** as scheduled minimum on a sufficient level.

During the semester there are two tests: the mid-term test in the 7th week and the end-term test in the 14th week. Students have to sit for the tests

Grades: 0-49% fail (mark 1), 50-59% satisfactory (mark 2), 60-69 % average (mark 3), 70-84% good (mark 4), 85-100 excellent (mark 5), we use rounding up (e.g. 49.3% is satisfactory).

Person responsible for course: Dr. Patricia Szokol, assistant professor, PhD
Lecturer: Prof. Dr. István Fazekas, university professor, DSc Dr. Patricia Szokol, assistant professor, PhD

Title of course: Multivariate Analysis **Code**: TTMME0403	**ECTS Credit points:** 3

Type of teaching, contact hours
- lecture: 2 hours/week
- practice: -
- laboratory: -

Evaluation: exam

Workload (estimated), divided into contact hours:
- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: 22 hours
- preparation for the exam: 40 hours

Total: 90 hours

Year, semester: 1st year, 1st semester

Its prerequisite(s):

Further courses built on it: TTMME0904

Topics of course

Multivariate sample and its properties; principal component analysis; exploratory factor analysis; canonical correlation analysis; classification methods, cluster analysis; multidimensional scaling; support vector machines.

Literature

J. Izenman: Modern Multivariate Statistical Techniques. Regression, Classification and Manifold Learning, Springer, 2008.
N. H. Timm: Applied Multivariate Analysis, Springer, 2002.
B. Everitt, T. Hothorn: An Introduction to Applied Multivariate Analysis with R, Springer, 2011.
D. Zelterman: Applied Multivariate Statistics with R, Springer, 2015.

Schedule:

1st week

Multivariate sample and its empirical characteristics. Wishart distribution. Multivariate normal sample.

2nd week

Maximum-likelihood estimation of parameters of a multivariate normal sample. Hotelling’s T-square test.

3rd week

Principal component analysis, properties of principal components.

4th week

Sample principal components. Scree plot, examples.

5th week

Fundamentals of exploratory factor analysis.

6th week

Estimation of parameters and testing of hypotheses in factor models. Factor rotation.

7th week

Canonical correlation analysis. Estimation of canonical factors. *8th week* Classification methods: maximum-likelihood and Bayes' decision. Estimation methods. *9th week* Logistic regression. Nearest neighbour method. *10th week* Cluster analysis: hierarchical methods, k-means clustering. *11th week* Multidimensional scaling: classical solution. *12th week* Nonmetric scaling. The Shepard-Kruskal algorithm. *13th week* Fundamentals of support vector machines. *14th week* Case studies.
Requirements: - *for a signature* Attendance at **lectures** is recommended, but not compulsory. - *for a grade* The course ends in an **oral examination**, where the knowledge of practical applications is a fundamental requirement.
Person responsible for course: Dr. Sándor Baran, associate professor, PhD
Lecturer: Dr. Sándor Baran, associate professor, PhD

Title of course: Multivariate Analysis **Code**: TTMMG0403	**ECTS Credit points:** 2

Type of teaching, contact hours
- lecture: -
- practice: -
- laboratory: 2 hours/week

Evaluation: mid-semester grade

Workload (estimated), divided into contact hours:
- lecture: -
- practice: -
- laboratory: 28 hours
- home assignment: 32 hours
- preparation for the final test: -

Total: 60 hours

Year, semester: 1st year, 1st semester

Its prerequisite(s):

Further courses built on it: -

Topics of course

Fundamentals of R; multivariate sample and its properties; principal component analysis; exploratory factor analysis; canonical correlation analysis; classification methods, cluster analysis; multidimensional scaling; support vector machines

Literature

B. Everitt, T. Hothorn: An Introduction to Applied Multivariate Analysis with R, Springer, 2011.
D. Zelterman: Applied Multivariate Statistics with R, Springer, 2015.

Schedule:

1st week

Fundamentals of R, commands, data structures.

2nd week

Functions in R. Packaging.

3rd week

Multivariate sample, descriptive statistics.

4th week

Data visualization.

5th week

Principal component analysis with R. Case studies.

6th week

Exploratory factor analysis with R. Case studies.

7th week

Canonical correlation analysis. Case studies.

8th week

Classification methods: linear and quadratic discriminant analysis. Case studies.

9th week

Logistic regression. Case studies.

10th week

Cluster analysis: hierarchical methods. Dendrograms, icicle plots. Case studies.

11th week

K-means clustering. Case studies.

12th week

Multidimensional scaling: classical solution. Case studies.

13th week

Nonmetric scaling. The Shepard-Kruskal algorithm. Case studies.

14th week

Fundamentals of support vector machines. Case studies.

Requirements:

- *for a grade*

Attendance of **laboratories** is compulsory. The course ends in a **practical test.**

Score	Grade
0-14	fail (1)
15-18	pass (2)
19-22	medium (3)
23-26	good (4)
27-30	excellent (5)

If the score of the test is below 15, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

Person responsible for course: Dr. Sándor Baran, associate professor, PhD

Lecturer: Dr. Sándor Baran, associate professor, PhD

Title of course: Option pricing **Code**: TTMME0404	**ECTS Credit points:** 3

Type of teaching, contact hours
- lecture: 2 hours/week
- practice: -
- laboratory: -

Evaluation: exam

Workload (estimated), divided into contact hours:
- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 62 hours

Total: 90 hours

Year, semester: 1st year, 1st semester

Its prerequisite(s): -

Further courses built on it: -

Topics of course

The students get to know about the fundamental derivatives and their roles, the fundamentals of the mechanism of derivatives markets, the principles of pricing derivatives, the principle of arbitrage and how to apply it for pricing, some classical models and problems and methods related to their fitting and applications.

Literature

Compulsory:
- Hull, J. C.: Options, Futures and Other Derivatives, Pearson, 10th edition, 2018.

Recommended:
- Musiela, M. and Rutkowsky, M.: Martingale Methods in Financial Modelling, 2nd edition, Springer, 2005.

Schedule:

1st week

Basic notions. Derivatives and their categories.

2nd week

Futures, forward contracts, standard options. Payoffs, profit. Examples.

3rd week

Notion of arbitrage. Pricing of futures. Forward price.

4th week

Differences of futures and forward contracts, pricing of special cases, examples.

5th week

Properties of option prices (factors affecting option prices, upper and lower bounds).

6th week

Put-call parity, early exercise. Elementary trading strategies (involving a single option and a stock).

7th week

Trading strategies involving options (spreads, combinations). *8th week* Option pricing in binary markets. Pricing of European options, hedging strategies, arbitrage-free valuation. *9th week* Binary and binomial markets. Pricing of American optionns. *10th week* Introduction to continuous time models. Volatility, the basics of Black-Scholes markets. *11th week* The Black-Scholes formula, and its applications, implied volatility. *12th week* Classification of risks. Basics of market risk management. *13th week* Greeks, delta hedging. *14th week* Estimation of option prices, approximations.
Requirements: The students get a grade based on a written exam. Grades: 0-49% fail (mark 1), 50-59% satisfactory (mark 2), 60-69 % average (mark 3), 70-84% good (mark 4), 85-100 excellent (mark 5), we use rounding up (e.g. 49.3% is satisfactory).
Person responsible for course: Dr. József Gáll, associate professor, PhD
Lecturer: Dr. József Gáll, associate professor, PhD

Title of course: Option pricing **Code**: TTMMG0404	**ECTS Credit points:** 2

Type of teaching, contact hours
- lecture: -
- practice: 2 hours/week
- laboratory: -

Evaluation: mid-semester grade

Workload (estimated), divided into contact hours:
- lecture: -
- practice: 28 hours
- laboratory: -
- home assignment: 16 hours
- preparation for the exam: 16 hours

Total: 60 hours

Year, semester: 1st year, 1st semester

Its prerequisite(s): -

Further courses built on it: -

Topics of course

The students get to know about the fundamental derivatives and their roles, the fundamentals of the mechanism of derivatives markets, the principles of pricing derivatives, the principle of arbitrage and how to apply it, some classical models and problems and methods related to their fitting and applications.

Literature

Compulsory:
- Hull, J. C.: Options, Futures and Other Derivatives, Pearson, 10th edition, 2018.

Recommended:
- Musiela, M. and Rutkowsky, M.: Martingale Methods in Financial Modelling, 2nd edition, Springer, 2005.

Schedule:

1st week

Derivatives and their categories.

2nd week

Futures, forward contracts, standard options. Payoffs, profit. Examples.

3rd week

Notion of arbitrage. Pricing of futures. Forward price.

4th week

Pricing of futures and forward contracts, special cases.

5th week

Examples of arbitrage. Properties of option prices (factors affecting option prices, upper and lower bounds).

6th week

Put-call parity, early exercise. Elementary trading strategies (involving a single option and a stock).

7th week

Trading strategies involving options (spreads, combinations).

8th week

Option pricing in binary markets. Pricing of European options, hedging strategies, arbitrage-free valuation.

9th week

Binary and binomial markets. Pricing of American optionns.

10th week

Introduction to continuous time models. Volatility, the basics of Black-Scholes markets.

11th week

The Black-Scholes formula, and its applications, implied volatility.

12th week

Classification of risks. Basics of market risk management.

13th week

Greeks, delta hedging.

14th week

Estimation of option prices, approximations.

Requirements:
The students get a grade based on an end-term test, which contains numerical exercises, questions from practice.Grades: 0-49% fail (mark 1), 50-59% satisfactory (mark 2), 60-69 % average (mark 3), 70-84% good (mark 4), 85-100 excellent (mark 5), we use rounding up (e.g. 49.3% is satisfactory).

Person responsible for course: Dr. József Gáll, associate professor, PhD

Lecturer: Dr. Bernadett Aradi, assistant professor, PhD

Title of course: Financial mathematics I **Code**: TTMME0405	**ECTS Credit points:** 3

Type of teaching, contact hours

- lecture: 2 hours/week
- practice: -
- laboratory: -

Evaluation: exam

Workload (estimated), divided into contact hours:

- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 62 hours

Total: 90 hours

Year, semester: 1st year, 2nd semester

Its prerequisite(s): -

Further courses built on it: **TTMME0406**

Topics of course

Discrete time models of stock markets and options, pricing of options, risk measures, coherent measures, Value at Risk, Expected Shortfall, operational risk and its models based on compound distributions. Markowitz's mean-variance portfolio analysis, CAPM.

Literature

Compulsory:

Gáll, J., G. Pap and M. v. Zuijlen (2003): "Option Theory", https://gyires.inf.unideb.hu/mobiDiak/Gall-Jozsef/Option-theory/op2.pdf

Recommended:

- Harry H. Panjer: Operational Risk: Modeling Analytics, Wiley, 2006.
- Musiela, M. and Rutkowsky, M.: Martingale Methods in Financial Modelling, 2nd edition, Springer, 2005.

Schedule:

1st week

Conditional expected value, martingales, related properties and theorems.

2nd week

Financial assets markets, derivatives. Discrete time markets, basic notions.

3rd week

Arbitrage.

4th week

Arbitrage.

5th week

Market completeness.

6th week

Fundamental theorems of option pricing.

7th week
Further option pricing theorems and cases.
8th week
Basic properties of risk measures, Value at Risk.
9th week
Basic properties of risk measures, Expected shortfall.
10th week
Operational risk. Compound distributions, AMA models and related estimations.
11th week
Mean-variance portfolio analysis.
12th week
Mean-variance portfolio analysis.
13th week
CAPM.
14th week
Summary of models, limitations of the models, discussion on the application.

Requirements:
The students get a grade based on an oral exam that includes the theoretical results (theorems, models, proofs) discussed in the term. .

Person responsible for course: Dr. József Gáll, associate professor, PhD

Lecturer: Dr. József Gáll, associate professor, PhD

Title of course: Financial mathematics I **Code**: TTMMG0405	**ECTS Credit points:** 2

Type of teaching, contact hours

- lecture: -
- practice: 2 hours/week
- laboratory: -

Evaluation: mid-semester grade

Workload (estimated), divided into contact hours:

- lecture: -
- practice: 28 hours
- laboratory: -
- home assignment: 16 hours
- preparation for the exam: 16 hours

Total: 60 hours

Year, semester: 1st year, 2nd semester

Its prerequisite(s): -

Further courses built on it: -

Topics of course

Discrete time models of stock markets and options pricing, risk measures, coherent measures, Value at Risk, Expected Shortfall, operational risk and its models based on composite distributions. Markowitz-type mean-variance portfolio analysis, CAPM.

Literature

Compulsory:

Gáll, J., G. Pap and M. v. Zuijlen (2003): "Option Theory", https://gyires.inf.unideb.hu/mobiDiak/Gall-Jozsef/Option-theory/op2.pdf

Recommended:

Harry H. Panjer: Operational Risk: Modeling Analytics, Wiley, 2006.

Musiela, M. and Rutkowsky, M.: Martingale Methods in Financial Modelling, 2nd edition, Springer, 2005.

Schedule:

1st week

Conditional expected value, martingales, related main theorems, properties.

2nd week

Markets of financial assets, derivatives. Discrete time markets, basic notions.

3rd week

Arbitrage.

4th week

Arbitrage.

5th week

Market completeness.

6th week

Fundamental theorems of option pricing.

7th week
Option pricing, further markets and cases.

8th week

Basic properties of risk measures, Value at Risk.

9th week

Basic properties of risk measures, Expected shortfall.

10th week

Operational risk. Models based on compound distributions (AMA) and related estimations.

11th week
Mean-variance portfolio analysis.

12th week
Mean-variance portfolio analysis.

13th week

CAPM.

14th week

Summary, discussion on the application of the models at issue.

Requirements:
The students get a grade based on an end-term test, which contains numerical exercises, questions from practice.

Grades: 0-49% fail (mark 1), 50-59% satisfactory (mark 2), 60-69 % average (mark 3), 70-84% good (mark 4), 85-100 excellent (mark 5), we use rounding up (e.g. 49.3% is satisfactory).

Person responsible for course: Dr. József Gáll, associate professor, PhD

Lecturer: Dr. Bernadett Aradi, assistant professor, PhD

Title of course: Introduction to Finance **Code**: TTMME0901	**ECTS Credit points:** 5
Type of teaching, contact hours - lecture: 2 hours/week - practice: 2 hours/week - laboratory: -	
Evaluation: exam	
Workload (estimated), divided into contact hours: - lecture: 28 hours - practice: 28 - laboratory: - - home assignment: 30 - preparation for the exam: 64 hours Total: 150 hours	
Year, semester: 2nd year, 1st semester	
Its prerequisite(s):	
Further courses built on it:	

Topics of course

Basic notions of finances and financial markets, time value of money, methods of calculating present value, other fundamental financial statements, financial statement frauds based on financial and market data, bonds and shares and basic methods of the pricing, internal rate of return, elementary questions on investment.

Literature

Compulsory:

Brealey, R. and Myers, S.: Principles of Corporate Finace, Concize Edition, McGraw Hill Higher Education, 2010.

Recommended:

Ross, S. A. - Westerfield, R. W. - Jordan, B. D.: Essentials of Corporate Finance, Mcgraw-Hill/Irwin, 2007.
Block, B. S.-Hirt, G. A.: Foundations of Financial Management, Mcgraw-Hill/Irwin, 2001.
Brigham, E. F. - Ehrhardt, M .C.: Financial Management, Theory and Practice, Harcourt College Publishers, 2002.

Schedule:

1st week

Basic (introductory) notions of finance.

2nd week

Financial markets, the role of the financial manager, financial tasks in a corporation.

3rd week

Cash flows, the time value of money.

4th week

Net present value and its applications.

5th week Annuities, perpetuities, compounding conventions. *6th week* Bonds and bond markets. *7th week* Valuation of bonds. *8th week* Stocks and stock markets. *9th week* Valuation of stocks. *10th week* NPV versus other criteria for financial decision making. *11th week* Internal rate of return, rate of return calculations. *12th week* Project analysis, investment decisions based on NPV. *13th week* The analysis of financial statements by financial ratios. *14th week* Financial ratios and their applications.
Requirements: The student can choose a 'two part' exam. In this case the results of the two test papers are included in the final grade (50%-50%). The first test of the 'two part' exam will be in the middle of the semester, whereas the second will take place at the end of the semester or in the first exam week. The tests include both theoretical questions and practical exercises. Further exams (for those who do not choose the two part exam opportunity or those who fail it) will be 'one part' exams (in the exam period), i.e. all chapters covered in the course will be required. The 'two part' exam cannot be repeated partially (i.e. only one part of it cannot be rewritten), only the whole exam can be rewritten in the exam period (as a 'one part' exam). The students may miss at most 3 seminars. In case of missing more than 3 seminars the seminar is not completed, hence the course is not completed. For this, a class attendance list will be made each week, which can be signed by the students only in the first 10 minutes of the seminar. To complete the seminar requirements the students are given some home assignments in the seminars which are discussed in the next seminars. Grades: 0-49% fail (mark 1), 50-59% satisfactory (mark 2), 60-69 % average (mark 3), 70-84% good (mark 4), 85-100 excellent (mark 5), we use rounding up (e.g. 49.3% is satisfactory).
Person responsible for course: Dr. József Gáll, associate professor, PhD
Lecturer: Dr. József Gáll, associate professor, PhD

Title of course: Microeconomics **Code**: TTMME0902	**ECTS Credit points:** 5

Type of teaching, contact hours

- lecture: 2 hours/week
- practice: 2 hours/week
- laboratory: -

Evaluation: exam

Year, semester: 1st year, 2nd semester

Its prerequisite(s): -

Further courses built on it: TTMME0903 Macroeconomics

Topics of course

The methodology of microeconomics, consumer theory, production theory and costs, profit-maximization on the competitive and monopoly market, welfare consequences of the monopoly.

Literature

Compulsory:

Besanko, David – Breautigam, Ronald R.: Microeconomics. Third Edition (International Student version). John Wiley and Sons, Inc., New York, 2008.

Besanko, David – Breautigam, Ronald R.: Microeconomics. Study Guide. Third Edition. John Wiley and Sons, Inc., New York, 2008.

Recommended:

Schedule:

1st week

Principles of microeconomics, equilibrium analysis – graphical treatment

2nd week

Price elasticity and other elasticities

3rd week

Consumer preferences and utility

4th week

The budget constraint

5th week

Consumer choice

6th week

Individual demand, consumer surplus and market demand

7th week

Production function

8th week

Costs

9th week

Cost-minimization

10th week Perfect competition I *11th week* Perfect competition II, long-run supply *12th week* Monopoly *13th week* The welfare economics of monopoly *14th week* Summary
Requirements: The exam is a written test which will be evaluated according to the following grading schedule: 0 - 50% – fail (1) 50%+1 point - 63% – pass (2) 64% - 75% – satisfactory (3) 76% - 86% – good (4) 87% - 100% – excellent (5)
Person responsible for course: Prof. Dr. Judit Kapás, university professor, PhD
Lecturer: Prof. Dr. Judit Kapás, university professor, PhD

Title of course: Econometrics **Code**: TTMME0904	**ECTS Credit points: 4**

Type of teaching, contact hours

- lecture: 2 hours/week
- practice: -
- laboratory: 1 hour/week

Evaluation: exam

Workload (estimated), divided into contact hours:

- lecture: 28 hours
- practice: -
- laboratory: 14 hours
- home assignment: 18 hours
- preparation for the exam: 60 hours

Total: 120 hours

Year, semester: 2nd year, 1st semester

Its prerequisite(s): TTMME0403

Further courses built on it: -

Topics of course

Topics of econometrics. Regression models: the OLS estimate, goodness-of-fitting, indices, hypothesis testing. Autocorrelation, multicollinearity. Dummy and truncated variables. Simultaneous econometrics models. Regression models for time series. Case studies. Regression models in R.

Literature

- G. S. Maddala, K. Lahiri: Introduction to Econometrics. 4th Edition. Wiley, 2009.
- R. Ramanathan: Statistical Methods in Econometrics. Academic Press, 1993.
- W. H. Greene: Econometric Analysis. 7th Edition.Pearson, 2012.
- C. Kleiber, A. Zeileis: Applied Econometrics with R, Springer, 2008.

Schedule:

1st week

Topics and history of econometrics. Elements of econometric models. Statistics with R.

2nd week

Simple linear regression, estimation of parameters, confidence intervals. Simple linear regression with R.

3rd week

Testing of hypotheses and analysis of variance in simple linear regression models. Nonlinear models.

4th week

Multiple linear regression models. Partial and multiple correlations. Multiple linear regression models with R.

5th week

Testing of hypotheses and goodness of fit in linear models. Case studies.

6th week

Model building, tests of stability. Case studies.

7th week

Heteroskedasticity. Implementation of various tests for heteroscedasticity in R. *8th week* Autocorrelation. Case studies. *9th week* Multicollinearity. Case studies. *10th week* Dummy variables. Logit and probit models. Case studies. *11th week* Simultaneous equation models. Case studies. *12th week* Regression models for time series. Case studies. *13th week* Case studies. *14th week* Project presentations.
Requirements: - *for a signature* Attendance of **lectures** is recommended, but not compulsory. Attendance of **laboratories** is compulsory. Students have to present an individual project. - *for a grade* The course ends in an **oral examination**, where the knowledge of practical applications is a fundamental requirement.
Person responsible for course: Dr. Sándor Baran, associate professor, PhD
Lecturer: Dr. Sándor Baran, associate professor, PhD

Title of course: Financial accounting **Code**: TTMME0905	**ECTS Credit points: 5**
Type of teaching, contact hours - lecture: 2 hours/week - practice: 2 hours/week - laboratory:	
Evaluation: exam	
Year, semester: 2nd year, 2nd semester	
Its prerequisite(s): -	
Further courses built on it: -	

Topics of course Notion of public accountancy. Steps in the accounting process. Accounting system, practice of public accountancy. International Financial Reporting Standards (IFRS). The content of financial statements and their presentation.
Literature

Schedule: *1st week* *2nd week* *3rd week* *4th week* *5th week* *6th week* *7th week* *8th week* *9th week* *10th week* *11th week* *12th week* *13th week* *14th week*
Requirements:
Person responsible for course: Kornél Tóth, senior assistant professor
Lecturer: Kornél Tóth, senior assistant professor

Title of course: Game theory **Code**: TTMME0208	**ECTS Credit points:** 3

Type of teaching, contact hours

- lecture: 2 hours/week
- practice: -
- laboratory: -

Evaluation: exam

Workload (estimated), divided into contact hours:

- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 62 hours

Total: 90 hours

Year, semester: 2nd year, 2nd semester

Its prerequisite(s): -

Further courses built on it: -

Topics of course

The normal form of non-cooperative games. The notion and existence of Nash equilibrium. The best response mapping. Fixed point theorems in game theory. Analysis of finite games, strictly dominated strategies, bi-matrix representation of finite two-player games. Mixed extension of finite games. Two-player zero-sum games, matrix games. Symmetric games. Games in extensive form. Combinatorial games, Grundy's games, Grundy numbering. Cooperative games, the value of the coalition. Nash's model of bargaining.

Literature

Compulsory:

- J. H. Conway: On Numbers and Games, Academic Press, 1976. ISBN 1568811276
- Martin J. Osborne: An Introduction to Game Theory, Oxford University Press, 2003. ISBN 0195128958

Recommended:

- Kim C. Border: Fixed point theorems with application to economic and game theory, Cambridge University Press, Cambridge UK, 1985. ISBN 0-521-38808-2
- J. von Neumann, O. Morgenstern: Theory of games and economic behavior, Princeton University Press, Princeton, New Jersey, 1944. ISBN 978 0 691 13061 3

Schedule:

1st week

The normal form of non-cooperative games. Strategies and strategy profiles. Utilities. The notion of Nash equilibrium. Strategically equivalent games. Bi-matrix representation of finite 2-player games.

2nd week

Finite games. Iterative elimination of strictly dominated actions.

3rd week

Transposable equilibrium points. Strictly competitive 2-player games. The value of the game.

4th week

Zero-sum games with 2 players. Equilibrium and the mini-max principle. Equilibrium strategies in symmetric zero-sum games with 2 players.

5th week

Sufficient conditions for the existence of Nash equilibrium. The best response mapping.

6th week

Extension of finite games through mixed strategies. Existence of (symmetric) Nash equilibrium.

7th week

Matrix games.

8th week

Extensive games. Decision tree. Sets of imperfect information.

9th week

Combinatorial game theory. Game of nim. Grundy numbers, the mex function. Direct sums.

10th week

Infinite games: the Banach–Mazur game (with intervals).

11th week

Finite matching problems I: Stable redistribution of properties. Construction of the solution in terms of graph representation.

12th week

Finite matching problems II: Algorithms for stable marriages.

13th week

Coalitions. Examples, valuation of coalitions.

14th week

Bargaining games with 2 players. Nash solution.

Requirements:

- *for a signature*

Attendance at **lectures** is recommended, but not compulsory.

- *for a grade*

The course ends in an oral **examination**. Exam topics are identical to those of the individual lectures. The grade is based on the presentation of the designated exam topic and the answers to the questions (on various topics) of the examiner.

Solving theoretical problems (posed during lectures) before or during the exam is taken in consideration as answer to non-basic exam questions (like proofs of theorems or lemmas).

Person responsible for course: Dr. Zoltán Boros, associate professor, PhD

Lecturer: Dr. Zoltán Boros, associate professor, PhD

Title of course: Game theory **Code**: TTMMG0208	**ECTS Credit points:** 2

Type of teaching, contact hours
- lecture: -
- practice: 2 hours/week
- laboratory: -

Evaluation: mid-semester grade

Workload (estimated), divided into contact hours:
- lecture: -
- practice: 28 hours
- laboratory: -
- home assignment: 24 hours
- preparation for the test: 8 hours

Total: 60 hours

Year, semester: 2nd year, 2nd semester

Its prerequisite(s): -

Further courses built on it: -

Topics of course

The normal form of non-cooperative games. The notion and existence of Nash equilibrium. The best response mapping. Fixed point theorems in game theory. Analysis of finite games, strictly dominated strategies, bimatrix representation of finite two-player games. Application of the game theoretic approach to simple market models (duopoly, oligopoly). Mixed extension of finite games. Two-player zero-sum games, matrix games. Games in extensive form. Combinatorial games, Grundy's games, Grundy numbering. Cooperative games, the value of the coalition. Finite matching problems.

Literature

Compulsory:
- J. H. Conway: On Numbers and Games, Academic Press, 1976. ISBN 1568811276
- Martin J. Osborne: An Introduction to Game Theory, Oxford University Press, 2003. ISBN 0195128958

Recommended:
- Kim C. Border: Fixed point theorems with application to economic and game theory, Cambridge University Press, Cambridge UK, 1985. ISBN 0-521-38808-2

J. von Neumann, O. Morgenstern: Theory of games and economic behavior, Princeton University Press, Princeton, New Jersey, 1944. ISBN 978 0 691 13061 3

Schedule:

1st week

The normal form of non-cooperative games. Strategies and strategy profiles. Utilities. The notion of Nash equilibrium. Examples. Bi-matrix representation of finite 2-player games.

2nd week

Finite games. Iterative elimination of strictly dominated actions.

3rd week

Discrete and continuous sharing games (heritage, crazy drivers).

4th week

Zero-sum games with 2 players. Equilibrium and the mini-max principle. Examples.

5th week

The best response mapping and the existence of Nash equilibrium. Application of the game theoretic approach to simple market models (duopoly, oligopoly).

6th week

Extension of finite games through mixed strategies.

7th week

Matrix games.

8th week

Extensive games. Decision tree. Deterministic and partially random examples.

9th week

Combinatorial game theory. Game of nim. Grundy numbers, the mex function. Direct sums.

10th week

Infinite games: the Banach–Mazur game (with intervals).

11th week

Finite matching problems I: Stable redistribution of properties. Construction of the solution in terms of graph representation.

12th week

Finite matching problems II: Algorithms for stable marriages.

13th week

End-term test.

14th week

Examples, valuation of coalitions.

Requirements:

- for a signature

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence. In case of further absences, a medical certificate needs to be presented. Missed practice classes should be made up for at a later date, to be discussed with the tutor. Active participation is evaluated by the teacher in every class. If a student's behaviour or conduct doesn't meet the requirements of active participation, the teacher may evaluate his/her participation as an absence because of the lack of active participation in class.

At the end of the semester there is a test in the 13th week. Students have to sit for the test.

- for a grade

The **seminar grade** is based on the result of the **end-term test**. Excellent contributions to practice classes may be taken into consideration by the tutor with extra points.

Based on the score of the test (and the extra points received during the semester), the grade for the seminar is given according to the following table:

Score (%)	Grade
0–49	fail (1)
50–59	pass (2)
60–74	satisfactory (3)
75–87	good (4)
88–100	excellent (5)

If the score of the test is below 50%, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.
Person responsible for course: Dr. Zoltán Boros, associate professor, PhD
Lecturer: Dr. Zoltán Boros, associate professor, PhD

Title of course: Macroeconomics **Code**: TTMME0903	**ECTS Credit points:** 5

Type of teaching, contact hours
- lecture: 2 hours/week
- practice: 2 hours/week
- laboratory: -

Evaluation: exam

Year, semester: 2nd year, 1st semester

Its prerequisite(s): TTMME0902

Further courses built on it: -

Topics of course

Central problems in macroeconomics. Principles of measuring aggregates: economic cycle and the GDP, nominal and real GDP, applications of GDP, the GDP-deflator and the consumer price index, measuring unemployment. Economy in the long run: equilibrium of the goods market, equilibrium of the factor market and the distribution of income, theories of natural unemployment. Importance of money and inflation: the functions of money and the money supply, quantity theory of money, money demand, costs of inflation. Short run models of economy: the Keynesian cross, the IS-LM model, models of aggregate supply and aggregate demand. Relation between short term and long term deductions: the expectations-augmented Philips curve and the Friedman and Modigliani-type theory of consumption functions.

Literature

Compulsory:

Mankiw, Gregory: Macroeconomics. Sixth Edition. Worth Publisher, New York, 2007.
Kaufman, Roger T.: Student Guide and Workbook for Use with Macroeconomics. Worth Publisher, New York, 2007.

Recommended:

Williamson, Stephen D. (2014). Macroeconomics. Fifth (International) Edition, Pearson

Schedule:

1st week
The fundamental questions of macroeconomics. The data of macroeconomics: production and income.
Mankiw, pp. 1-15, Kaufman, pp. 1-8., Mankiw, pp. 16-30., Kaufman, pp. 9-18.

2nd week
The data of macroeconomics: inflation and unemployment. The economy in the long run: production and the division of income.
Mankiw, pp. 30-43., Kaufman, pp. 19-29., Mankiw, pp. 44-59., Kaufman, pp. 30-45.

3rd week:
The economy in the long run: demand and equilibrium on market for goods and services.
Mankiw, pp. 59-75., Kaufman, pp. 46-58.

4th week
Money supply.
Mankiw, pp. 76-83, 510-517., Kaufman, pp. 59-64, 357-367.

5th week
The quantity theory of money, and the Fisher effect. The demand for money, the costs of inflation.
Mankiw, pp. 83-94., Kaufman, pp. 64-68., Mankiw, pp. 95-111., Kaufman, pp. 68-79.

6th week
The natural rate of unemployment: job search. The natural rate of unemployment: real-wage rigidity
Mankiw, pp. 159-165., Kaufman, pp. 111-122., Mankiw, pp. 165-184., Kaufman, pp. 111-122.

7th week
Introduction to economic fluctuations.
Mankiw, pp. 252-277., Kaufman, pp. 159-174.

8th week
Aggregate demand: the Keynesian Cross and the IS curve.
Mankiw, pp. 278-292., Kaufman, pp. 175-198., Mankiw, pp. 292-298., Kaufman, pp. 199-204.

9th week
Short-run equilibrium in the IS-LM model.
Mankiw, pp. 299-313., Kaufman, pp. 205-220.

10th week
The IS-LM model as a theory of aggregate demand I.
Mankiw, pp. 313-328., Kaufman, pp. 220-244.

11th week
The IS-LM model as a theory of aggregate demand II.
Mankiw, pp. 313-328., Kaufman, pp. 220-244.

12th week
Aggregate supply.
Mankiw, pp. 373-380., Kaufman, pp. 267-282.

13th week
The Phillips curve.
Mankiw, pp. 385-400., Kaufman, pp. 282-290.

14th week
Summary

Requirements:
The exam is a written test which will be evaluated according to the following grading schedule:
0 - 50% – fail (1)
50%+1 point - 63% – pass (2)
64% - 75% – satisfactory (3)
76% - 86% – good (4)
87% - 100% – excellent (5)

Person responsible for course: Dr. Pál Czeglédi, associate professor, PhD

Lecturer: Dr. Pál Czeglédi, associate professor, PhD

Title of course: Insurance mathematics **Code**: TTMME0407	**ECTS Credit points:** 3

Type of teaching, contact hours
- lecture: 2 hours/week
- practice: --
- laboratory: -

Evaluation: exam

Workload (estimated), divided into contact hours:
- lecture: 28 hours
- practice:
- laboratory: -
- home assignment: 20
- preparation for the exam: 42 hours

Total: 90 hours

Year, semester: 2nd year, 2nd semester

Its prerequisite(s):

Further courses built on it: -

Topics of course

Notion of insurance, classification of insurances, classical non-life insurance models, methods for determining total loss, related regression and statistical questions. Pricing. Life and reinsurances, annuity calculation, pricing of life insurances.

Literature

Compulsory:

Straub, Erwin: Non-life Insurance Mathematics, Springer-Verlag, 1980.
Mikosch, Thomas: Non-life Insurance Mathematics, Springer, Berlin, Heidelberg, New York, 2006.

Recommended:

Schedule:

1st week

Basic notions of insurance and insurance contracts.

2nd week

Non-life insurance models for the aggregate claim.

3rd week

Recursion methods for the total claim amount, the De Pril algorithm.

4th week

Berry-Essen inequalities and estimation of the distribution of the total claim by normal distribution.

5th week

Moment generating functions, generator functions, Laplace transform.

6th week

Compound distributions. Distributions for the number of claims. (a,b,0) distributions.

7th week

Fitting methods for the distribution of claim numbers.

8th week

Fitting problems for the distribution of the individual claims. The role of inflation and retention.

9th week

Methods for the calculation of the total claim amount, Panjer's algorithm.

10th week

Prices and fees. Further problems in non-life insurance.

11th week

Basics of life insurance.

12th week

Perpetuity and annuity based calculations.

13th week

Reinsurance contracts. Main types.

14th week

Summary, further examples.

Requirements:

The students are given home assignments during the semester, it is required to solve them for the signature.
The course can be completed by an oral exam at which the students are given both practical exercises and theoretical questions.

Person responsible for course: Dr. Bernadett Aradi, assistant professor, PhD

Lecturer: Dr. József Gáll, associate professor, PhD,
Dr. Bernadett Aradi, assistant professor, PhD

Title of course: Financial mathematics II **Code**: TTMME0406	**ECTS Credit points:** 3

Type of teaching, contact hours

- lecture: 2 hours/week
- practice: -
- laboratory: -

Evaluation: exam

Workload (estimated), divided into contact hours:

- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: -
- preparation for the exam: 62 hours

Total: 90 hours

Year, semester: 2nd year, 1st semester

Its prerequisite(s): **TTMME0405**

Further courses built on it: -

Topics of course

Utility theory, expected utility, axioms and criticism in related literature. Risk aversion and its measuring, optimal portfolios. Contionuous time shares and interest-rate models, analysis of arbitrage-freeness, pricing of shares, bonds and interest-rate derivatives and models.

Literature

Compulsory:

Gáll, J., G. Pap and M. v. Zuijlen (2005): "An introduction to portfolio management", https://gyires.inf.unideb.hu/mobiDiak/Gall-Jozsef/An-introductionto-portfolio-management/portf-en.pdf

Musiela, M. and Rutkowski, M.: Martingale Methods in Financial Modeling, Springer-Verlag, Berlin, Heidelberg, 2005.

Brigo, D. and Mercurio, F.: Interest Rate Models - Theory and Practice: With Smile, Inflation and Credit, Springer, Berlin, Heidelberg New York, 2006

Recommended:

Björk, T.: Arbitrage Theory in Continuous Time, Oxford University Press, Oxford/New York, 1998.

Schedule:

1st week

Utility theory, axioms.

2nd week

Expected utility and axioms.

3rd week

Expected utility, fundamental theorems.

4th week

Risk aversion and its measures.

5th week

Expected utility based portfolio optimisation, demand of financial assets. *6th week* Continuous time financial market models, basic notions. *7th week* Change of measure in continuous time, absence of arbitrage. *8th week* Black-Scholes market, and Black-Scholes formula. *9th week* Further models and problems for option pricing in continuous time. *10th week* Bond market, yield curves, interest rates. *11th week* Arbitrage free family of bond prices. Fundamental theorems. *12th week* Change of measure in bond markets, forward measure. *13th week* Basics of short interest rate models. *14th week* Problems in specific short rate models.
Requirements: The course can be completed by an oral exam that contains theoretical questions (theorems, proof, models).
Person responsible for course: Dr. József Gáll, associate professor, PhD
Lecturer: Dr. József Gáll, associate professor, PhD

Title of course: Finite Geometries and Coding Theory **Code**: TTMME0303	**ECTS Credit points: 3**

Type of teaching, contact hours
- lecture: 2 hours/week
- practice: -
- laboratory: -

Evaluation: exam

Workload (estimated), divided into contact hours:
- lecture: 28 hours
- practice: -
- laboratory: -
- home assignment: 22 hours
- preparation for the exam: 40 hours

Total: 90 hours

Year, semester: 1st or 2nd year, 2nd semester

Its prerequisite(s): -

Further courses built on it: -

Topics of course

Finite incidence structures: projective and affine planes, Galois geometry. Combinatorial properties of finite projective planes. Arcs and ovals. Finite projective planes and algebraic structures. Finite projective and affine planes over a field. Examples of combinatorial point sets on finite projective plane. Further incidence structures: block design and Steiner-system. Applications of finite geometry in coding theory.

Literature

Compulsory:

A. Beutelspacher: Projective Geometry – From Foundations to Applications, Cambridge, 1998.

Recommended:

J. W. P. Hirschfeld: Projective Geometries Over Finite Fields, Oxford, 1998.

D. R. Hughes, F. C. Piper: Projective Planes, Springer, 1973.

S. E. Payne: Topics in Finite Geometry, 2007.

Schedule:

1st week

Affine and projective planes.

2nd week

Affine and projective planes over finite fields. Collineation groups of field planes.

3rd week

Cyclic planes and difference sets.

4th week

Polarities and conics. Hermite-curves in projective planes over finite fields.

5th week

Blocking sets. Subplanes.

6th week

Arcs, ovals, hyperovals. The Theorem of Segre.

7th week

Coordinating of projective planes. Connections of the algebraic properties of the coordinating structure and the geometric properties of the projective plane.

8th week

Latin squares.

9th week

Higher dimensional projective spaces. Galois geometries.

10th week

Block designs.

11th week

Steiner Triple Systems and Steiner Quadruple Systems.

12th week

Basics of coding theory. Constructions of codes from finite planes.

13th week

MDS codes and arcs of finite projective planes.

14th week

Applications of finite geometries in cryptography.

Requirements:

Only students who have signature from the practical part can take part of the exam. The exam is written. The grade is given according to the following table:

Score	Grade
0-49	fail (1)
50-62	pass (2)
63-74	satisfactory (3)
75-86	good (4)
87-100	excellent (5)

Person responsible for course: Dr. Zoltán Szilasi, senior assistant lecturer, PhD

Lecturer: Dr. Zoltán Szilasi, senior assistant lecturer, PhD

Title of course: Finite Geometries and Coding Theory **Code**: TTMMG0303	**ECTS Credit points: 2**

Type of teaching, contact hours
- lecture: -
- practice: 2 hours/week
- laboratory: -

Evaluation: mid-semester grade

Workload (estimated), divided into contact hours:
- lecture: -
- practice: 42 hours
- laboratory: -
- home assignment: 18 hours
- preparation for the exam:

Total: 60 hours

Year, semester: 1st or 2nd year, 2nd semester

Its prerequisite(s): -

Further courses built on it: -

Topics of course

Finite incidence structures: projective and affine planes, Galois geometry. Combinatorial properties of finite projective planes. Arcs and ovals. Finite projective planes and algebraic structures. Finite projective and affine planes over a field. Examples of combinatorial point sets on finite projective plane. Further incidence structures: block design and Steiner-system. Applications of finite geometry in coding theory.

Literature

Compulsory:

A. Beutelspacher: Projective Geometry – From Foundations to Applications, Cambridge, 1998.

Recommended:

J. W. P. Hirschfeld: Projective Geometries Over Finite Fields, Oxford, 1998.

D. R. Hughes, F. C. Piper: Projective Planes, Springer, 1973.

S. E. Payne: Topics in Finite Geometry, 2007.

Schedule:

1st week

Minimal model of affine planes, Fano plane. Geometric construction of affine and projective planes over small fields.

2nd week

Analytic problems in projective planes over finite fields.

3rd week

Constructions of cyclic planes and difference sets.

4th week

Applications of finite affine and projective planes in solving combinatorical problems.

5th week

Examples of blocking sets.

6th week

Examples of arcs, ovals and hyperovals.

7th week

Ternary rings and quasifields – proofs of some simple properties.

8th week

Examples of quasifields.

9th week

Applications of Plücker coordinates.

10th week

Examples of block designs and inversive planes.

11th week

Constructions of Steiner Triple Systems.

12th week

Constructions of Steiner Quadruple Systems.

13th week

Constuctions of finite codes using finite geometries.

14th week

Test.

Requirements:

- for a signature

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence.

- for a grade

During the semester one test is written. The grade is given according to the following table:

Score	Grade
0-49	fail (1)
50-59	pass (2)
60-74	satisfactory (3)
75-84	good (4)
85-100	excellent (5)

Students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

Person responsible for course: Dr. Zoltán Szilasi, senior assistant lecturer, PhD

Lecturer: Dr. Zoltán Szilasi, senior assistant lecturer, PhD

Title of course: Fourier series **Code**: TTMME0206	**ECTS Credit points:** 4

Type of teaching, contact hours

- lecture: 2 hours/week
- practice: 1 hours/week
- laboratory: -

Evaluation: exam

Workload (estimated), divided into contact hours:

- lecture: 28 hours
- practice: 14 hours
- laboratory: -
- home assignment: 26 hours
- preparation for the exam: 52 hours

Total: 120 hours

Year, semester: 2nd year, 1st semester

Its prerequisite(s): -

Further courses built on it:

Topics of course

The interpolation theorems of Marcinkiewicz, classical and complex trigonometric systems, the theorems of Weierstrass, the density of trigonometric polynomials, the Riemann-Lebesgue lemma, Dirichlet kernels, Fejér kernels, norm convergence of Fejér means, the Calderon-Zygmund decomposition, Hilbert operator, Fejér-Lebesgue theorem, the Dini and the Lipschitz criteria for convergence, the norm convergence of Fourier partial sum operators, Fourier series with respect to Walsh systems.

Literature

Compulsory:-
Recommended:
N. K. Bary: A Treatise on Trigonometric Series, Elsevier, 2014.
A. Zygmund, Trigonometric Series Vol I., Cambridge University Press, 2002.

Schedule:

1st week The interpolation theorems of Marcinkiewicz.

2nd week The classical and complex trigonometric system, the approximation theorems of Weierstrass.

3rd week Trigonometric polynomials, and their density in Lebesgue spaces.

4th week The Riemann-Lebesgue lemma, the Dirichlet kernels and their fundamental properties,

5th week Fejér kernel functions and their fundamental properties.

6th week Norm convergence of Fejér means in various spaces.

7th week The Calderon-Zygmund decomposition lemma.

8th week The Hilbert operator and some of its properties.

9th week The maximal operator of the Fejér means and its quasi-locality.

10th week The Fejér-Lebesgue theorem with respect to almost everywhere convergence

11th week Riemann's first localization theorem, Dini and Lipschitz convergence criteria

12th week Partial sum operators of Fourier series, their uniform weak and strong type boundedness.

13th week Norm convergence of trigonometric Fourier series in Lebesgue spaces.

14th week Some convergence and divergence properties of other orthonormal systems, the Walsh system.

Requirements:

- for a signature
Attendance at **lectures** is recommended, but not compulsory.

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. A student can't make up any practice with another group. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence. In case of further absences, a medical certificate needs to be presented. Missed practice classes should be made up for at a later date, to be discussed with the tutor. Active participation is evaluated by the teacher in every class. If a student's behaviour or conduct doesn't meet the requirements of active participation, the teacher may evaluate his/her participation as an absence because of the lack of active participation in class.

During the semester there are two tests: the mid-term test in the 7th week and the end-term test in the 14th week. Students have to sit for the tests.

- for a grade
The course ends in an **examination**.

The minimum requirement for the average of the mid-term and end-term tests and also for the examination is 50%. The grade for the examination is given according to the following table, where the score is (X+Y+4Z)/6, where X, Y are the scores of the tests and Z is the score of the performance on the examination.

Score	Grade
0-49	fail (1)
50-61	pass (2)
62-74	satisfactory (3)
75-87	good (4)
88-100	excellent (5)

If the average of the scores of the tests is below 50, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

Person responsible for course: Prof. Dr. György Gát, university professor, DSc

Lecturer: Prof. Dr. György Gát professor, DSc